Competition for Listings

Thierry Foucault*  Christine A. Parlour †

February, 2000

Abstract

We develop a model in which two profit maximizing exchanges compete for IPO listings. They choose the listing fees paid by entrepreneurs wishing to go public and control the trading costs incurred by investors. All entrepreneurs prefer lower costs, however entrepreneurs differ in how they value a decrease in trading costs. Hence, in equilibrium, competing exchanges obtain positive expected profits by offering different execution costs and different listing fees. As a result, firms that list on different exchanges have different characteristics. The model has testable implications for the cross-sectional characteristics of IPOs on different quality exchanges and the relationship between the level of trading costs and listing fees. We also find that competition does not guarantee that exchanges choose welfare maximizing trading rules.

Keywords: Exchanges, Trading Costs, Listings, Competition.

*Dept of Finance, HEC, School of Management and CEPR, 1 rue de la Liberation, 78351 Jouy en Josas, France. Tel: (33) 1 39 67 94 11. Fax: (33) 1 39 67 70 85. Email: foucault@gwsmtp.hec.fr
†G.S.I.A., Carnegie Mellon University, Pittsburgh, PA, 15213, Email: parlourc@andrew.cmu.edu.

We are grateful to seminar participants at the AFA2000 meetings, ESSEC, INSEAD, G.S.I.A, Université Catholique de Louvain and to Yakov Amihud, Rick Green, Pekka Hietala, Burton Hollifield, Harald Han, Uday Rajan, Bryan Routledge, Neal Stoughton, S. Viswanathan for helpful comments. We also thank Marianne Demarchi and Solemne Thomas from the SBF for providing us with information regarding listing requirements in various exchanges. Funding from the Carnegie Bosch Institute is gratefully acknowledged. All errors are ours.
1 Introduction

The amount of money that a firm can raise, when it initially sells off claims to a cash flow, depends on the resale value of such claims and hence on the liquidity of the secondary market in which its shares will trade. So, there is a relationship between the characteristics of the marketplace offered by a particular exchange and firms’ decisions to list on the exchange. In this paper, we analyze how this relationship influences competing exchanges’ choice of trading rules.

To do so, we examine three distinct, but inter-related questions about the industrial organization of stock exchanges. First, with two different trading systems, how will firms trying to raise equity choose between such systems? Second, in the longer run, does competition induce exchanges to adopt similar or different trading systems and is this choice socially optimal? Finally, why do we observe such a diversity of trading systems? In the U.S.A., the Nasdaq is a celebrated broker/dealer market while the NYSE employs specialists. If one market form is inherently better at providing liquidity, why doesn’t a single market type dominate? We study these questions with a model in which:

- Profit maximizing exchanges choose (i) a per share trading cost incurred by investors when they trade in the secondary market (that we interpret as an outcome of trading rules/market microstructure) and (ii) set competitive listing fees incurred by entrepreneurs who list.

- Entrepreneurs, who own productive technologies, want to sell equity shares that are claims to the payoffs of these technologies. Entrepreneurs differ in the size (value) of their companies. Each chooses (i) where to list, (ii) how many shares to sell (what percentage ownership of the technology to cede) and (iii) the price at which shares are initially sold to the public.

- Investors purchase shares in the Initial Public Offerings conducted by the entrepreneurs. These investors may be hit by liquidity shocks that force them to liquidate their portfolios. For this reason, trading costs on the different exchanges influence the prices at which IPOs take place.

We view exchanges as simple profit maximizers. Their revenues depend on listings
in two ways. First, an exchange charges a listing fee to firms. Second, investors incur execution costs each time they trade listed shares. In practice, both source of revenues (trading and listing fees) are important to exchanges. In 1996, for North American stock exchanges, listing revenues accounted for 30.9% of total revenues whereas trading revenues accounted for 27.9% of total revenues.\(^1\) Hence, exchanges trade–off the number of listings they can attract with their trading technology against the listing fees they can charge. Clearly, this trade–off is resolved differently for different exchanges: listing fees on the NYSE are nearly an order of magnitude higher than those on the Nasdaq.\(^2\)

Investors, who anticipate high execution costs in the secondary market, pay less for IPOs issued on high trading cost exchanges. Entrepreneurs take this into account in their listing decision and thus, other things equal, prefer to list on the exchange with the lowest trading cost. Hence, in equilibrium, a low cost exchange can charge high fees and entrepreneurs balance better IPO prices against larger listing fees. As a consequence, competing exchanges strategically choose different trading technologies to soften competition for listings. Indeed, competing exchanges with different trading costs (one with strictly larger trading costs but lower listing fees), can co–exist. The implications are:

- Entrepreneurs with different characteristics choose to list on different exchanges. Hence, issue size, market value, and the elasticity of demand for shares in an IPO vary with trading cost.

- Listing fees are larger on a low trading cost exchange and the listing fee differential between two exchanges is proportional to the difference in execution costs.

- Social welfare is maximized when competing exchanges choose the lowest pos-

---

\(^1\)Source: FIBV Annual Report, 1997. Other revenue comes from additional services offered by Stock Exchanges such as market data dissemination, clearing and settlement services etc. Note that the revenue generated by these ancillary services is also partly proportional to the number of listings.

\(^2\)The maximum initial listing fee on Nasdaq is $50,000 versus $504,600 for the NYSE. Corwin and Harris (1998) estimate that for a firm issuing 5 million shares at $5, the initial listing fee represents 0.0375% of market value on Nasdaq against 0.1058% on the NYSE. It is also interesting to note that the share of listing revenue and trading revenue in total revenue varies across exchanges. Angel and Aggarwal (1996) estimate that 41% of the NYSE annual revenue in 1996 came from listing fees. For Nasdaq, listing fees represented only 20% of total revenue in the same year.
sible trading cost, but competition for listings between profit maximizing exchanges is not sufficient to guarantee this. In contrast, volume maximizing exchanges do choose the lowest trading cost market structure.

Our model of differentiation in trading technologies can be extended to explain minimum size requirements imposed by exchange listing standards. We show how exchanges optimally choose different minimum size requirements to soften competition for listings. We also show that there is a limited number of possible exchanges that can be profitably created, even when the cost of setting up an exchange is very small.

There are several distinctive features of our model. First, in our framework, there is no ‘clientele effect’ in listing location choice and no exogenous cost to moving between exchanges (e.g., due to national boundaries). Our analysis, therefore, has relevance for situations where different exchanges compete for listings within the same country (as, for example, in the U.S. or in Canada). Second, as we have mentioned, we model exchanges as profit maximizers and consider trading rules as a variable of choice. Third, trading costs in the secondary market affect entrepreneurs’ listing decision. In line with this feature of our model, Amihud and Mendelson (1986) find that the cost of capital increases with trading costs. As trading rules imply a trading cost, their result creates a relationship between the choice of a listing location and the rules that govern secondary market trading. We are aware of only one empirical paper that examines listing choices between Nasdaq and NYSE (Corwin and Harris (1998)). They find that listing fees and execution costs do affect listing decisions. This is a feature of our model. Further, our theoretical implications on the cross-sectional characteristics of IPOs between two exchanges accord with their findings.

In our model, the listing decisions of firms are determined by exchanges’ market structures. This distinguishes the paper from extant literature in which other

---

3Clientele effects on both investors and firms have been offered as an explanation for the co-existence of different trading technologies. See Angel and Aggarwal (1997) and Harris (1990).

4Indeed firms pay attention to trading costs in their listing decision. For instance, Cowan et al. (1992) show that firms with larger spreads are more likely to leave Nasdaq in order to be listed on the NYSE.

5Most of the empirical literature on listing choice has focused on the price effects when firms delist from Nasdaq in order to list on the NYSE (Cowan et al. (1992), Grammatikos and Papaioannou (1986), Sanger and McConnell (1986), Baker (1996)). The findings in this empirical literature are not relevant for the present article as we focus on firms’ initial listing decisions.
exchange characteristics determine the listing decision. For instance, Huddart et al. (1999) consider the role of disclosure requirements. Angel and Aggarwal (1997) focus on the sponsorship services provided by brokers/dealers, whereas Chemmanur and Fulghieri (1999) consider the certification effect of listing standards. Gehrig, Stahl and Vives (1996) analyze the role of informational asymmetries between domestic and foreign investors for firms listing abroad. Furthermore, in contrast to these papers, listing fees are endogenous in our analysis and provide an incentive for exchanges to differentiate their trading systems.

The next section describes the model. Section 3 describes and characterizes the outcome of the IPO process. In Section 4 we characterize the equilibrium of the competition game between exchanges and we discuss the empirical implications in Section 5. In Section 6, we derive the policy implications of the model. Section 7 presents an extension to different types of listing requirements and allows for entry in the market for exchange services. Section 8 concludes. All the proofs are in the Appendix and a table of our notation appears at the end of the paper (just after the bibliography).

2 The Model

We consider an economy with three classes of agents: entrepreneurs, investors and exchanges. The timing of the game that these agents play is described in Figure 1. There are 5 dates. At date 1, two profit maximizing exchanges choose trading rules which generate a per share cost of trading. At date 2, they set listing fees. At date 3, each entrepreneur who owns a productive technology partially divests from his company by going public. To do so, he chooses an exchange on which to list and sell shares to investors in an IPO. Investors, who may have liquidity shocks, are willing to buy shares of this technology to transfer money across time. At date 4, some of the investors are hit by liquidity shocks and have to sell shares. At date 5, the firms’ payoffs are realized.

Entrepreneurs: There is a continuum of entrepreneurs, each of type \( t \) where \( t \) is uniform on \([\underline{t}, \bar{t}]\). An entrepreneur’s type is the expected per share payoff from his productive technology which will be realized at date 5. Each entrepreneur can issue
at most $N$ shares, therefore $tN$ is the total size of any firm. The average expected per share payoff in the population is $m$. We will sometimes refer to the dispersion of firms’ expected payoffs, $[\bar{t} - \underline{t}]$ which we denote $\sigma$. Hence, $\underline{t} = m - \frac{\sigma}{2}$ and $\bar{t} = m + \frac{\sigma}{2}$. As all cash flows are positive, $m > \sigma$.

Entrepreneurs derive utility from consumption at the IPO stage, date 3 ($C_3$) and date 5 ($C_5$). For each entrepreneur,

$$U(C_3, C_5) = C_3 + \delta C_5,$$

where $\delta < 1$ is the entrepreneur’s intertemporal preference parameter. In order to increase date 3 consumption, entrepreneurs go public and sell some of their shares or equivalently, a percentage of ownership of the future proceeds from their productive technology, to outside investors. Thus, entrepreneurs in our model can be thought of as venture capitalists or original owners who want to ‘cash in’ by selling off shares.\(^6\) Entrepreneurs maximize utility by choosing (i) whether to go public, (ii) the listing location and (iii) the issue size and the IPO price.

**Investors:** A continuum of potential investors indexed by $s$ where $s$ is uniform on $[0, 1]$ bid for shares in the IPOs. An investor of type $s$ is hit by a liquidity shock at time 4 with probability $s$ and with probability $(1 - s)$ she is hit by a shock at time 5. At time 3, the IPO stage, each investor knows her probability (type). An investor that is hit by a liquidity shock must liquidate her portfolio. Hence, the expected utility\(^7\) of investor $s$ is:

$$E[U_s(C_3, C_4, C_5)] = C_3 + sEC_4 + (1 - s)EC_5.$$  \(^2\)

Each investor has sufficient wealth at the IPO stage (date 3) so that she is never wealth constrained.\(^8\) However, with no date 4 or 5 endowment she wishes to buy shares from entrepreneurs to transfer wealth across time. She can also invest in a

\(^6\)This is, indeed, one of the reason for which firms go public. See, for instance, Ellingsen and Rydqvist (1997) or Pagano and Zingales (1998).

\(^7\)Our specification of investors’ preferences is similar to Gorton and Pennacchi (1990) or Bolton and Von Thadden (1998).

\(^8\)Introducing wealth constraints for investors or entrepreneurs creates technical complexities but it does not qualitatively change our results.
riskless asset whose rate of return, per period, is normalized to zero. At date 4, investors that are hit by liquidity shocks sell off their portfolios. Trades are executed on the exchange on which the shares are listed. For each firm, investors that are not hit by liquidity shocks buy the shares sold at date 4 at a price, \( p^{SEC}(t) = t \), equal to the firm’s expected per share payoff. Without affecting the results, we assume that trading costs are entirely borne by sellers.

**Exchanges:** Exchanges are profit maximizers who derive profits from two sources: listing fees that they charge to entrepreneurs and revenues from trading in the secondary market. There are two exchanges, Exchange 1 and Exchange 2. At date 1, the exchanges simultaneously choose a trading technology. A trading technology is a specific set of trading rules, the outcome of which is a per share execution cost in the secondary market, \( c_j, j = 1, 2 \).

In the market microstructure literature, different trading rules are associated with different levels of trading costs. For simplicity, we do not explicitly model the relationship between trading rules and trading costs. Rather, we assume that exchanges choose a trading technology associated with a trading cost or equivalently a level of quality. There are two possible trading technologies: (i) a high cost technology, \( c_H \), which is of low quality \( q_L = \frac{1}{c_H} \) or (ii) a low cost technology, \( c_L \) which is of high quality \( q_H = \frac{1}{c_L} \).

The trading cost is not necessarily a trading fee. That is, costs faced by investors are not necessarily completely recovered by the exchanges. We denote by \( \gamma \geq 0 \) the fraction of the total trading cost \( c_j \) that is recovered by an exchange.

A value of \( \gamma \) less than 1 can be interpreted in two ways. First, the market microstructure literature is replete with examples in which there are sources of execution costs that are partially controlled by the exchange (through the design of its trading rules) but that do not directly generate revenues for the exchange. For example, consider the minimum price variation (tick size). Tick size is chosen by an exchange and

---

9 We assume that the entrepreneurs cannot short sell the riskless asset. Otherwise, the difference in intertemporal preference parameters between entrepreneurs and investors precludes the existence of an equilibrium in the market for the riskless asset since all the agents are risk neutral.

10 Bertrand competition among the buyers insures that this price is indeed the equilibrium price in the secondary market. Note that the entrepreneurs have no utility for consumption at date 4. Thus they do not sell or short-sell shares at this date. If the entrepreneurs could consume at date 4, the equilibrium in the secondary market would exist only if we assume that short sales are forbidden.
creates a wedge between the fair value of the asset and the price at which investors can buy or sell the asset. This wedge is a transaction cost but is not recovered by the exchange. Rather, it allows liquidity suppliers (e.g. limit order traders or dealers) to capture rents.\textsuperscript{11} Second, a fraction of the order flow may be routed away from the exchange on which a firm has listed. This will result in a lower $\gamma$ for the exchange.\textsuperscript{12}

At date 2, exchanges simultaneously set a listing fee,\textsuperscript{13} $F_j$, that must be paid by a firm if it lists on the exchange. In sum, an exchange, say $j$, is characterized by $(c_j, F_j)$, \textit{i.e.}, a specific bundle of trading cost and listing fee.

Let $T_j \subseteq [t, \bar{t}]$ be the subset of entrepreneurs who list on Exchange $j$. Let $Vol(t, q_j)$ be the expected trading volume in the secondary market for the shares issued by an entrepreneur of type $t$. We normalize the marginal cost of an additional listing to zero. Therefore, the total expected profit of Exchange $j$ is:

$$
\Pi_j(T_j) = F_j \Pr[t \in T_j] + \gamma c_j E[Vol(t, q_j) \mid t \in T_j].
$$

A listing generates two types of revenue for the exchange. First, the exchange obtains the listing fee. Second, the exchange obtains revenue proportional to the trading volume in this listing. Exchange $j$ chooses $(c_j, F_j)$ to maximize its total expected profit.

## 3 The Initial Public Offering Process

Consider an entrepreneur who has decided to go public on Exchange $j$. We derive the number of shares that are sold by the entrepreneur in the IPO (the ‘issue size’) and the price at which he sells these shares (‘the IPO price’). Then, we compute the

\textsuperscript{11}Subrahmanyam and Chordia (1995) provide a model in which the minimum price variation enables dealers to capture strictly positive expected profits at the expense of liquidity demanders. Madhavan (1992) compares trading costs (due to asymmetric information) in two markets with different trading rules: an order-driven market and a quote-driven market. He shows that trading costs differ in these two trading mechanisms. This illustrates that an exchange can control execution costs, even when these costs do not directly accrue to the exchange.

\textsuperscript{12}For instance, in the U.S., shares listed on the NYSE are traded on regional exchanges and OTC. Empirically, the original listing location retains the lion’s share of trading, however. Hasbrouck (1995) reports that for the Dow 30 stocks, the NYSE executes on average 84.5% of the daily volume.

\textsuperscript{13}Exchanges charge two types of listing fees. An \textit{entry fee} that is paid up-front when the firm initially lists and a \textit{continuation fee} that is paid annually. For our model, this distinction is not relevant.
utility benefit to the entrepreneur from going public and we relate it to the liquidity of the exchange.

Our model of price formation in the IPO is a stylized rendering of a book building. First, investors truthfully report their valuation for the issue. After observing the schedule of bids, the entrepreneur decides what fraction, \( (1 - \alpha) \), of his \( N \) shares he will sell. Given the investors' demand, this determines an IPO price. All investors with a valuation greater than or equal to the IPO price get an equal number, \( N_{ds} \), of shares.\(^\text{14}\) The entrepreneur retains the residual.

Let \( V(s, t, c_j) \) be the per share valuation of an investor with type \( s \), for a firm with an expected payoff \( t \), that is listed on Exchange \( j \). This is the maximum that such an investor will bid in the IPO. It is immediate that:

\[
V(s, t, c_j) = t - sc_j, \tag{4}
\]

the expected payoff per share minus the investor’s expected trading cost. Thus, investors’ required rate of return increases with the size of the trading cost.\(^\text{15}\)

The higher the probability of a shock, the larger the expected trading costs faced by an investor in the secondary market, hence, the less she is willing to pay for the stock. Therefore, \( V(s, t, c_j) \) decreases with the probability of a liquidity shock, \( s \). So, that if an investor of type \( s^* \) buys in the IPO, then all investor types with \( s < s^* \) participate as well. Hence, if the entrepreneur wants to sell a fraction \( (1 - \alpha) \) of his \( N \) shares then the investor who is indifferent between buying and not, \( s^* \), is such that:

\[
\int_0^{s^*} N_{ds} = [1 - \alpha]N. \tag{5}
\]

Clearly, \( s^* = (1 - \alpha) \). To sell \( (1 - \alpha)N \) shares, the IPO price\(^\text{16}\) must be equal to

---

\(^\text{14}\)The specific allocation rule we consider enables us to get closed form solutions for the equilibrium. The crucial assumption is that no investor can buy more than a small fraction of all the shares in a given issue. This could be because investors choose to be well diversified or, alternatively, entrepreneurs want to tap the wealth of different classes of investors (institutional investors, small investors) to avoid any large stake being assembled by a single investor (Brennan and Franks (1997)).

\(^\text{15}\)In the model, as investors are risk-neutral, their required return is independent of the size of their stake.

\(^\text{16}\)The IPO outcome can be implemented with a uniform price auction in which (i) the entrepreneur announces the size of the issue, (ii) investors post bids for a fixed quantity, (ii) the IPO price is chosen so as to equate supply and demand and (iii) all the orders from investors with a bid larger than or equal to the IPO price are filled. As there is a continuum of investors, it is optimal for each investor to post a bid equal to her valuation.
the valuation of the marginal investor of type $s^*$. Thus,

**Lemma 1**: The inverse demand curve faced by an entrepreneur of type $t$ who lists on exchange $j$ is

$$p^{IPO}(\alpha, t, c_j) = V(s^*, t, c_j) = t - c_j(1 - \alpha).$$  \hfill (6)

Figure 2 depicts this function for different levels of trading costs in the secondary market. Observe that the IPO price is decreasing in the number of shares, $(1 - \alpha)N$, that the entrepreneur sells. So, the demand for IPOs is not perfectly elastic. In our model, this is because the larger the number of shares the entrepreneur sells, the larger the set of investors the entrepreneur must tap into. Hence, the marginal investor in the IPO has a higher probability of a liquidity shock and consequently a lower valuation for the shares.

Let $\Delta U(\alpha, t, c_j)$ be the increase in the expected utility (gross of the listing fee) of the entrepreneur who goes public on Exchange $j$ and who retains a proportion $\alpha$ of all the shares. This is made up of two parts: first the IPO proceeds and second the loss in future cash flows from having sold shares. Or,

$$\Delta U(\alpha, t, c_j) = N(1 - \alpha)p^{IPO}(\alpha, t, c_j) - \delta(1 - \alpha)Nt$$

$$= (1 - \delta)N(1 - \alpha)t - (1 - \alpha)^2Nc_j,$$  \hfill (7)

where the last line follows from the IPO price (Equation (6)).

Trade occurs at the IPO stage because each entrepreneur has a smaller valuation for a unit of future consumption than each investor. The former value future consumption at $\delta$ and the latter at 1. Accordingly, the benefit from going public (the first term of Equation (8)) increases in $(1 - \delta)$, the size of the gains from trade between investors and entrepreneurs. If the secondary market were perfectly liquid ($c_j = 0$), it would be optimal for the entrepreneur to sell an arbitrarily large number of shares because the difference in valuation between the entrepreneur and the public is independent of each investors’ stake. When the secondary market is illiquid ($c_j > 0$), as the entrepreneur issues more shares, the valuation of the marginal buyer in the IPO decreases. As a consequence, the number of shares that the entrepreneur sells
is bounded. Thus, the trading cost is a source of inefficiency since it prevents gains from trade between investors and entrepreneurs at the IPO stage.

The entrepreneur’s optimal retained stake, $\alpha^*$, maximizes his expected utility from going public. The solution to this problem determines the IPO price and the issue size.

**Lemma 2 (IPO price and Issue Size):** An entrepreneur of type $t$ that lists on Exchange $j$ (with quality $q_j$) sells $[1 - \alpha^*(t, q_j)]N$ shares to the public where

$$\alpha^*(t, q_j) = 1 - \frac{(1 - \delta)tq_j}{2},$$

(9)

at a price $p^{IPO} = \frac{(1 + \delta)t}{2}$. He obtains a maximal utility benefit of:

$$\Delta U(\alpha^*(t, q_j), t) = \frac{(1 - \delta)^2t^2Nq_j}{4}.$$

(10)

Given the number of shares an entrepreneur of type $t$ will optimally issue if he lists on Exchange $j$ and the probability that his shareholders will be hit by liquidity shocks, we can determine the expected trading volume in the secondary market.\(^17\)

**Lemma 3:** The expected trading volume in the secondary market of entrepreneur $t$’s shares listed on Exchange $j$ is:

$$Vol(t, q_j) = \frac{(1 - \delta)Ntq_j}{4}.$$  

(11)

Thus, the expected trading volume increases with the quality of the exchange on which a firm lists (or equivalently decreases with the per share trading cost paid by investors) and increases with the size of the firm, $t$. Hence, given entrepreneurs’ decision in IPOs and $T_j$, the set of firms who list on Exchange $j$, the expected profit of the exchange is just:

$$\Pi_j(T_j) = F_j \Pr[t \in T_j] + \gamma E\left[\frac{(1 - \delta)Nt}{4} \mid t \in T_j\right].$$

(12)

\(^{17}\)The optimal number of retained shares $\alpha^*(t, q_j)$ is strictly positive if $q_j$ is sufficiently small (the trading cost sufficiently large). For our results, we only require that the number of retained shares be bounded, which is guaranteed for any positive trading cost. In practice $\alpha < 0$ can be interpreted as a short sale in the IPO by the issuer.
An exchange’s trading cost does not directly enter the profit function. This is because if the trading cost per share increases, entrepreneurs optimally issue fewer shares which implies that expected trading volume decreases. So, the trading revenue is unchanged. However, trading costs are important to an exchange’s revenue because they affect the entrepreneurs’ listing decisions and hence the set $T_j$.

4 Competition for Listings and the choice of Trading Technologies.

In this section, we show that it can be optimal for exchanges to choose different technologies. To analyze this, we first take trading technologies as given and look at the listing fee sub-game in the two possible cases that can arise in equilibrium: either both exchanges choose the same technology or both exchanges choose a different technology. In the second case, we establish conditions under which the high cost (low quality) exchange attracts listings in equilibrium. In subsection 4.3 we use these results to establish that it can be optimal for two exchanges to choose different trading technologies. To simplify computations, we fix $\delta = 1/2$. This assumption does not qualitatively affect results since the parameter $\delta$ just determines the size of the gains from trade between investors and entrepreneurs. In fact, all our results are robust for all parameterizations for which there are gains from trade, or $\delta < 1$.

For a given pair of listing fees $(F_j, F_j')$, an entrepreneur of type $t$ lists on Exchange $j$ if this gives him the highest maximal utility benefit of going public net of listing fees or if:

$$\Delta U(\alpha^*(t, q_j), t) - F_j > \max[\Delta U(\alpha^*(t, q_j'), t) - F_j', 0].$$

(13)

The “max” reflects the fact that if $F_j \geq \Delta U(\alpha^*(t, q_j), t)$, the listing fee is larger than his maximal utility benefit, and the entrepreneur prefers not to go public. If an entrepreneur does go public, using the maximal utility benefit from going public (Equation (10)), Equation (13) becomes

$$\frac{N[ q_j - q_j']^2}{16} > F_j - F_j'. $$

(14)
Thus, entrepreneurs face a trade–off between the quality and the listing fee of an exchange. To understand this trade–off, consider the maximal utility benefit of going public, \( \Delta U(\alpha^*(t, q_j), t) \). Three of its properties are crucial for our analysis of competition between exchanges.

**Lemma 4**: The maximal utility benefit of going public on Exchange \( j \), \( \Delta U(\alpha^*(t, q_j), t) \), increases with the size of the firm and increases in the quality of the exchange, or

1. \( \frac{d\Delta U}{dt} > 0 \),
2. \( \frac{d\Delta U}{dq} > 0 \).

Further, large firms benefit more from increases in exchange quality than small firms or,

3. \( \frac{d^2\Delta U}{dt dq} > 0 \).

Observe that all firms prefer lower trading costs, but larger firms benefit more for a given improvement in exchange quality. The entrepreneurs of these firms are more sensitive to the size of the trading cost since they sell more shares to the public. Thus, they are willing to pay larger listing fees for a better trading technology. For this reason, in equilibrium, firms that list on exchanges with different execution costs (different qualities) have different characteristics, as shown in Section 5.

### 4.1 A Benchmark: Competition for Listings with Identical Trading Technologies.

If the two exchanges have the same technology, \( (q_j = q_{j'}) \), Equation (14) becomes

\[
F_j < F_{j'}.
\]

Thus, all firms list on the exchange with the lowest listing fee. If the fees are equal, we assume that firms randomize between exchanges with probability \( 1/2 \). Entrepreneurs can also choose not to list, so we consider a fee less than the maximum utility benefit to entrepreneurs or \( F \leq \Delta U(\alpha^*(t, q_j), t) \). At such a fee, all entrepreneurs are willing to go public. Using Equation (12) and the randomization rule, the profit of Exchange \( j \) is just
\[ \Pi_j(F_j, F'_j) = \begin{cases} \frac{1}{2}F + \frac{N\gamma m}{16} & \text{if } F_j = F'_j = F, \\ F + \frac{N\gamma m}{8} & \text{if } F_j = F < F'_j, \\ 0 & \text{otherwise.} \end{cases} \]

Undercutting is profitable as long as \( F > -\frac{N\gamma m}{8} \). Thus, in equilibrium both exchanges choose a listing fee equal to \( F^* = -\frac{N\gamma m}{8} \) and obtain zero expected profit, just as in the traditional model of Bertrand price competition. If \( \gamma > 0 \), then exchanges derive some revenue from the trading costs and the listing fee is a subsidy. As trading in the secondary market generates revenue for them, they are willing to subsidize firms that go public since listings are necessary to create trading volume.

**Proposition 1**: If two exchanges competing in listing fees have the same trading technology so that \( q_1 = q_2 \) then (1) all entrepreneurs go public and (2) each exchange makes zero expected profits.

Observe that in this case, as firms randomize between the exchanges, the cross-sectional characteristics of firms on either exchange are the same.

### 4.2 Competition for Listings With Different Trading Technologies.

Suppose now that the two exchanges have different trading technologies, so that trading costs are different between the two markets. We assign index 1 to the low trading cost (high quality) exchange (i.e. \( c_1 = c_L \) so that \( q_1 = q_H = 1/c_L \) and \( c_2 = c_H \) so that \( q_2 = q_L = 1/c_H \)). We denote the difference in quality between the two exchanges by \( \Delta q = q_1 - q_2 \), and the difference in listing fees by \( \Delta F = F_1 - F_2 \).

As all entrepreneurs prefer low cost (high quality) exchanges (Lemma 4), for the high cost (low quality) exchange to attract any listings, it must charge lower listing fees, or \( F_2 < F_1 \). In this case, entrepreneurs trade-off a larger utility benefit, gross of the listing fee, if they go public on the high quality (low cost) exchange against the higher listing fee charged by this exchange. If the difference in listing fees between the exchanges is sufficiently large relative to the difference in quality, some firms will choose to list on the low quality exchange. From Lemma 4 we know that large firms are more willing to pay for increases in exchange quality than small firms. This property naturally leads to a sorting condition on firms who do go public.
Lemma 5: Consider two exchanges: if \( q_1 > q_2 \) and \( F_1 > F_2 \), then there exists a firm type \( t_c \), such that for \( t > t_c \), the entrepreneur prefers to list on the high quality (low cost) Exchange 1 and for \( t \leq t_c \), the entrepreneur prefers to list on the low quality (high cost) Exchange 2, where \( t_c = \max[\bar{t}, t^*] \) and

\[
t^*(F_1, F_2) = 4\sqrt{\frac{\Delta F}{N\Delta q}}.
\] (15)

Thus, \( t^*(F_1, F_2) \) is the entrepreneur type who is indifferent between listing on the low quality exchange and the high quality (low cost) exchange. If this type is larger than the smallest firm size \( (t^* > \bar{t}) \), then both exchanges will have a positive market share.

From Equation (15), both the listing fee and the quality of an exchange are important determinants of the choice of a listing location by a firm and hence the market share of an exchange. This feature of our model is consistent with Corwin and Harris (1998) who find that differences in listing costs and execution costs between the NYSE and Nasdaq are important factors in the initial listing decision of firms.

To give a clearer indication of the game, we define the profit functions of the two exchanges and the possible equilibria which can obtain in the listing fee sub-game. To describe the profit functions, we first calculate the listing fees for which each firm captures the whole market (‘exclusionary’ fee) and the fees for which each firm gets no listings at all (the ‘opt out’ fee).

Let \( F_{j\min}(F'_j) \) be the exclusionary fee for Exchange \( j \) when its competitor chooses a fee equal to \( F'_j \). Recall, from Lemma 5 that Exchange 1’s market share is \([t^*, \bar{t}]\), whereas Exchange 2’s market share is \([\bar{t}, t^*] \). So, the exclusionary fee for Exchange 1 is the one so that \( t^* = \bar{t} \) or Exchange 2’s market share is zero. For Exchange 2 the exclusionary fee is one such that \( t^* = \bar{t} \) or Exchange 1’s market share is zero. From the definition of \( t^* \):

\[
F_{1\min}(F_2) = F_2 + \frac{N(\Delta q)t^2}{16},
\]

\[
F_{2\min}(F_1) = F_1 - \frac{N(\Delta q)t^2}{16}.
\]

For each exchange there is also a listing fee (the ‘opt out fee’) such that the exchange attracts no listings at all, for a given listing fee of its competitor. Let \( F_{j\max}(F'_j) \) be this fee for Exchange \( j \). Using the definition of \( t^* \) again:
\[ F_1^{\text{max}}(F_2) = F_2 + \frac{N(\Delta q)t^2}{16}, \]
\[ F_2^{\text{max}}(F_1) = F_1 - \frac{N(\Delta q)t^2}{16}. \]

We can now determine (using Equation (12)), Exchange 1’s expected profit:

\[
\Pi_1(F_1, F_2) = \begin{cases} 
0 & \text{if } F_1 \geq F_1^{\text{max}}, \\
F_1 + \frac{\gamma N m}{8} & \text{if } F_1 \leq F_1^{\min}, \\
(t^*-\tilde{t})F_1 + (\frac{\gamma N}{16})(t^*-\tilde{t})^2 & \text{otherwise},
\end{cases}
\]

In the same way, Exchange 2’s expected profit is:

\[
\Pi_2(F_1, F_2) = \begin{cases} 
0 & \text{if } F_2 \geq F_2^{\text{max}}, \\
F_2 + \frac{\gamma N m}{8} & \text{if } F_2 \leq F_2^{\min}, \\
(\frac{t^*-\tilde{t}}{\sigma})F_2 + (\frac{\gamma N}{16})(t^*-\tilde{t})^2 & \text{otherwise},
\end{cases}
\]

where \(\tilde{t} = \max[t, 4\sqrt{F_2/\Delta q}]\). The entrepreneur with type \(\tilde{t}\) is just indifferent between listing on Exchange 2 or not going public.

Using the profit functions, we can determine the reaction functions of each exchange. To do so, let \(F_j^{\text{int}}\) be the listing fee for Exchange \(j\) that maximizes profits on \((F_j^{\min}, F_j^{\max})\) and let \(\Pi_j^{\text{int}}\) be this maximum profit. The reaction function of Exchange \(j\) is:

\[
F_j^r(F_j') = \begin{cases} 
F_j^{\min} & \text{if } \Pi_j(F_j', F_j) \geq \max[\Pi_j^{\text{int}}, 0], \\
F_j^{\max} & \text{if } \max[\Pi_j(F_j', F_j), \Pi_j^{\text{int}}] < 0, \\
F_j^{\text{int}} & \text{otherwise}.
\end{cases}
\]

In the two first cases, the optimal response of Exchange \(j\) is a corner solution of its maximization problem, whereas in the last case it is an interior solution. A pair of listing fees \((F_1^*, F_2^*)\) is a Nash equilibrium iff \(F_1^* = F_1^r(F_2^*)\) and \(F_2^* = F_2^r(F_1^*)\).

In what follows we focus on pure strategy equilibria in the listing fee game. A sufficient condition for existence of such equilibria is that \(2(\Delta q)\tilde{t} \geq \gamma\). We will always assume that this is the case.\(^{18}\)

\(^{18}\)Suppose that this condition is violated, so that \(\gamma\) is very large. Then, each exchange garners a large revenue from each transaction. Each therefore, has an incentive to undercut in listing fees to get extra listings. If the quality differential is small relative to this revenue gain, this effect prevents
Recall, that Exchange 1 is of high quality (low cost). It is intuitive, therefore that in equilibrium, it will never be driven out of the market. Further, observe that if Exchange 1 is the only exchange serving the market, then all entrepreneurs go public. To see this, recall that the maximal utility benefit to an entrepreneur of going public is always positive. So, if there is an equilibrium in which only Exchange 1 serves the market and some entrepreneurs do not go public then Exchange 2 can offer a positive listing fee, attract the unserved entrepreneurs and make profits. Hence:

**Lemma 6**:

1. There is no equilibrium in which Exchange 1 does not attract listings.
2. In equilibrium, if only Exchange 1 attracts listings then all firms go public.
3. The equilibrium listing fees are either $(F_{int}^1, F_{int}^2)$ (both exchanges attract listings) or $(F_{min}^1, F_{max}^2)$ (only Exchange 1 attracts listings and it attracts all the listings).

Is the exclusionary pricing policy $(F_1 = F_{min}^1)$ always optimal for Exchange 1? or are there conditions under which a low quality (high cost) exchange has a positive market share in equilibrium? Indeed, the high quality (low cost) exchange might be better off charging a larger fee that only attracts entrepreneurs with the highest willingness to pay. Such a cream–skimming policy leaves some room for the low quality Exchange 2, which can then charge a listing fee that attracts the remaining entrepreneurs. This policy turns out to be optimal for Exchange 1 if the dispersion of entrepreneur types is sufficiently large or if $\sigma > \sigma^c$, where $\sigma^c \equiv \frac{2m}{5}$.

**Proposition 2**: Consider an economy with two exchanges in which the quality differential is strictly positive ($\Delta q > 0$). If the dispersion of entrepreneur types is sufficiently large so that $\sigma > \sigma^c$ then both exchanges attract listings in equilibrium.

Listings are split between the two exchanges if the dispersion of firm size is sufficiently large. In this case, the low trading cost exchange is better off specializing in the existence of a pure strategy equilibrium. Technical details of this condition appear in Lemma 8 in the appendix.
relatively large firms rather than choosing a very low fee that would prevent Exchange 2 from attracting any listings. Note, that the proposition holds even if \( \gamma = 0 \), i.e., if exchanges do not capture any revenue from trading in the secondary market. This cream–skimming policy, when the dispersion of firms sizes becomes sufficiently small, is suboptimal for Exchange 1, as stated in the next proposition.

**Proposition 3**: Consider an economy with two exchanges in which the quality differential is strictly positive \((\Delta q > 0)\). If the dispersion of entrepreneurs is sufficiently small so that \( \sigma \leq \sigma^c \) then in equilibrium the high quality (low cost) Exchange 1 attracts all the listings.

It is natural to relate the dispersion of firm types to the size of the economy. If the economy is sufficiently large, that is if \( \sigma > \sigma^c \), then two different exchanges is the natural outcome of competition in listings. If, by contrast, \( \sigma \) is sufficiently small, then the exchange with the highest quality serves as a natural monopoly. This suggests that as economies grow, we should expect more exchanges with different trading technologies. This intuition is further explored in Section 7.2.

Alternatively, one can interpret the dispersion of entrepreneur types as a function of the outside opportunities for raising capital. If there are few opportunities outside equity markets for raising capital, that is if \( \sigma \) is large, then we predict differentiated equity markets. If, however, there are many and well–developed avenues for raising funds, so that \( \sigma \) is small in the economy, then we predict an equilibrium with a single exchange.

### 4.3 Long Term Competition in Trading Technologies.

Given the previous subsections, it is natural to ask: if exchanges choose their trading technology, what technologies will they choose? That is, will exchanges choose to differentiate? Will one of them optimally choose a low quality (high cost)? This question is pertinent for regulators who are concerned with the optimal industrial organization of stock exchanges.

First, observe that the ex ante profits of firms in the different equilibria can be explicitly ranked. Define \( \Pi_L \) to be the expected profit of the low quality (high cost)
Proposition 4: Consider an economy with two exchanges who have chosen different trading technologies:

1. If $\sigma > \sigma^c$ so that both exchanges attract listings then both exchanges make strictly positive expected profits and the profit of the high quality (low cost) exchange is higher than the profit of the low quality (high cost) exchange. Or,

   \[ \Pi_H > \Pi_L > 0. \]

2. If $\sigma \leq \sigma^c$ then only the high quality (low cost) exchange attracts listings. Or,

   \[ \Pi_H > \Pi_L = 0. \]

When the exchanges choose the same trading technology, they get a zero expected profit (Proposition 1). Table 1 below is the matrix of exchanges’ expected profits as a function of their trading technology choices at date 1. Exchange $j$ is the row player and $j'$ is the column player. Payoffs are recorded as $(\pi_j, \pi_{j'})$.

<table>
<thead>
<tr>
<th></th>
<th>$c_H$</th>
<th>$c_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_H$</td>
<td>(0, 0)</td>
<td>$(\Pi_H, \Pi_L)$</td>
</tr>
<tr>
<td>$c_L$</td>
<td>$(\Pi_H, \Pi_L)$</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

**Table 1**

Clearly, there are two Nash equilibria in pure strategies, if $\sigma > \sigma^c$. Either Exchange $j$ chooses the low trading cost technology and Exchange $j'$ chooses the high trading cost technology or vice versa.\(^{19}\) If, however, $\sigma \leq \sigma^c$, it is a weakly dominant strategy for both exchanges to choose the low trading cost technology (because $\Pi_L = 0$). The next proposition summarizes this discussion.

Proposition 5: In an economy in which two exchanges can choose their trading technology:

\(^{19}\)There is also a mixed strategy equilibrium in which each exchange chooses the high trading cost technology with probability $\beta = \frac{\Pi_H}{\Pi_L + \Pi_H}$. 

19
1. If \( \sigma > \sigma^c \) then the outcome of the pure strategy Nash equilibrium is that each exchange chooses a different technology.

2. If \( \sigma \leq \sigma^c \) then there is a unique weakly dominant Nash equilibrium in which both exchanges choose the high quality (low cost) technology.

This proposition establishes that it can be optimal for an exchange not to provide a trading technology that guarantees the lowest execution costs in the secondary market.\textsuperscript{20} An exchange which differentiates itself softens price competition in listings. Therefore, the benefit to an exchange of a large trading cost is strategic in our model: it enables an exchange to earn positive profits. By contrast, if both exchanges choose the same trading technology, they differ only by listing fees. Hence, Bertrand competition drives expected profits to zero.

5 Testable Implications.

We have established a condition (\( \sigma > \sigma^c \)) under which two exchanges competing for listings in an economy will choose different trading technologies and both have positive market shares. In this case, our model has testable implications for IPOs on competing exchanges and the relationship between trading costs and listing fees on these exchanges.

5.1 Implications for IPOs

We have noted that exchanges should attract listings from firms with different sizes. Hence, firm characteristics (issue size, proceeds, market value) of IPOs’ on two exchanges with different execution costs under the same jurisdiction should differ in a systematic way. In particular:

\textbf{Corollary 1} : In any economy in which two exchanges with different trading technologies compete for listings, the expected proportion of original shares offered to the

\textsuperscript{20}This is socially inefficient. We discuss social optimality in Section 6.
public is larger for an IPO taking place on the high quality (low cost) Exchange 1 than
for an IPO taking place on the low quality (high cost) Exchange 2, or

\[ E[(1 - \alpha^*(t, q_1)) \mid \text{Lists on Exchange 1}] > E[(1 - \alpha^*(t, q_2)) \mid \text{Lists on Exchange 2}] \]

This suggests that the post IPO ownership structure of firms who list on the low
cost (high quality) exchange should be less concentrated than for firms that list on
the high cost (low quality) exchange.

**Corollary 2**: In any economy in which two exchanges with different trading tech-
nologies compete for listings:

1. The expected proceeds for an IPO taking place on the high quality (low cost)
   Exchange 1 are larger than for an IPO taking place on the low quality (high
cost) Exchange 2, or

\[ E[(1 - \alpha^*(t, q_1)) Np^{IPO} \mid \text{Lists on Exchange 1}] > E[(1 - \alpha^*(t, q_2)) Np^{IPO} \mid \text{Lists on Exchange 2}] \]

2. The expected market value of a firm listed on the high quality (low cost) Exchange
   1 is larger than for a firm listed on the low quality (high cost) Exchange 2, or

\[ E[(1 - \alpha^*(t, q_1)) Np^{SEC} \mid \text{Lists on Exchange 1}] > E[(1 - \alpha^*(t, q_2)) Np^{SEC} \mid \text{Lists on Exchange 2}] \]

Of course, if trading costs are identical for the two exchanges, our model does not
predict cross–sectional differences. In the U.S. it is well-documented that execution
costs are larger on Nasdaq (see for instance Huang and Stoll (1996)). Corwin and
Harris (1998) study initial listing choices for a sample of firms that conducted Initial
Public Offerings between 1991 and 1996, either on the NYSE and the Nasdaq. They
restrict their attention to firms that are eligible to list on both exchanges. They
document substantial differences between the IPOs that take place on the NYSE and
Nasdaq. In particular they find (Table 3, page 30) that the number of offered shares,
the offer proceeds and the post IPO market value of firms that list on the NYSE are
significantly larger than on Nasdaq. These findings are consistent with our corollaries.
Corollary 3: In any economy in which two exchanges with different trading technologies compete for listings, the inverse demand curve for IPOs is steeper for the high cost (low quality) exchange. Or,

\[
\frac{dp^{IPO}(\alpha, t, c_H)}{d(1 - \alpha)N} = -c_H < \frac{dp^{IPO}(\alpha, t, c_L)}{d(1 - \alpha)N} = -c_L
\]

Kandel et al. (1999) show empirically that the demand for shares in IPOs is not perfectly elastic. Our corollary further suggests that (1) execution costs should be a main determinant of the elasticity of the demand for shares in IPO and (2) that this elasticity should differ systematically between exchanges with different execution costs. Corollary 3 could be tested using the schedule of bids (the book) for IPOs taking place on Nasdaq and NYSE.

5.2 Implications on Listing Fees

A further set of predictions exists on the listing fees. First, according to the model, listing fees on the high quality (low cost) exchange must be greater than listing fees on the low quality (high cost) exchange. This property is consistent with stylized facts. In the U.S., empirical studies find that Nasdaq has higher execution costs than the NYSE. At the same time, listing fees (in absolute terms and as a percentage of market capitalization) are much higher on the NYSE than on the Nasdaq (see Corwin and Harris (1998)).

To generate further predictions, we provide explicit closed form solutions for listing fees when each exchange has positive market share (\(\sigma > \sigma^c\)), and all firms go public. In the model, competition will generate fees low enough so that all firms go public if the percentage difference in execution costs, \(a = \frac{c_H - c_L}{c_H}\), is less than 33% (this is a sufficient condition). Intuitively, if the difference in trading costs is small, then competition in listing fees is heightened. Hence, the fee of the high cost (low quality) exchange is sufficiently low so that even the smallest entrepreneur finds it worthwhile to go public. Until the end of this section, we therefore assume that \(a \leq 33\%\).

Proposition 6: In an economy in which two exchanges attract listings (\(\sigma > \sigma^c\)), and the percentage difference in trade execution costs is sufficiently small, (\(a \leq \frac{1}{3}\)),
the equilibrium listing fees are:

\[ F_1^* = [b_1(\sigma)\Delta q - b_0(\gamma)] N \]
\[ F_2^* = [b'_1(\sigma)\Delta q - b_0(\gamma)] N, \]

where

\[ b_1(\sigma) = \left( \frac{m}{5} + \frac{\sigma}{2} \right) \frac{m}{10} \]
\[ b'_1(\sigma) = \left( \frac{\sigma}{2} - \frac{m}{5} \right) \frac{m}{10} \]
\[ b_0(\gamma) = \frac{\gamma m}{10} \]

The following corollaries analyze the impact of a change in exogenous parameters on the listing fees given in the previous proposition.

**Corollary 4**: Differences in quality soften price competition so that the listing fee of each exchange increases with the difference in quality between the two exchanges. Or,

\[ \frac{dF_1^*}{d\Delta q} > 0, \]
\[ \frac{dF_2^*}{d\Delta q} > 0. \]

If exchanges compete by changing their trading system so that the quality differential decreases, then there should be a corresponding decrease in listing fees. Anecdotal evidence suggests that in recent years, the gap between NYSE and Nasdaq trading costs has declined. In line with our prediction, the NYSE has responded by capping listing fees. Note that for unilateral changes in exchange quality, the effect on listing fees depends on which exchange is changing its execution cost. If the high quality (low cost) exchange decreases its trading cost then the quality differential increases. In contrast, if the low quality (high cost) exchange decreases its trading cost then the quality differential decreases. Therefore, if Nasdaq decreases execution costs, listing fees should fall, but if the NYSE decreases execution costs, then listing fees should rise.
Corollary 5: The difference in listing fees is proportional to the quality differential. Or,

\[ F_1^* - F_2^* = \left( \frac{m^2}{25} \right) \Delta q. \]  

(16)

Note that \( \Delta q \) is proportional to the percentage difference in execution costs. Thus, the difference in listing fees should increase when the difference in execution costs increases.\(^{21}\)

Corollary 6: The higher the revenues from trading volume, the lower the listing fee of each exchange. Or,

\[ \frac{dF_1^*}{d\gamma} < 0, \]
\[ \frac{dF_2^*}{d\gamma} < 0. \]

For each exchange, the opportunity cost of losing one listing, which includes the loss of trading fees generated by this listing, increases as \( \gamma \) increases. It follows that exchanges compete more aggressively for listings. As in the benchmark case, the two exchanges can even offer subsidies (negative listing fees) if \( \gamma > 0 \). Interestingly, this corollary suggests that the existence of third parties competing for order flow (so that \( \gamma \) is lower) may be a way for exchanges to credibly commit to charging high listing fees, whereas a movement to recover trading costs for an exchange signals a willingness to compete aggressively on listing fees. In the U.S., regional exchanges and Electronic Communication Networks (ECNs) such as Instinet, capture part of the order flow in shares listed on exchanges. We predict that an increase in the fraction of the order flow in listed shares captured by these trading venues could lead to an increase in listing fees charged by NYSE and Nasdaq.

\(^{21}\)Execution costs depend both on trading organization and on firms’ characteristics. Our model and its implications focus on the component of execution cost, which is exchange specific. One way to estimate this component is to measure total trading costs for matched samples of firms listed on different exchanges. The difference between the average trading costs for these samples can be ascribed to structural differences in the organization of the exchanges and can be used as a proxy for \((c_H - c_L)\) in our model. This is the method used in Huang and Stoll (1996) or Affleck-Graves et al. (1994) for instance.
6 Policy Implications and Regulation.

Should Stock Exchanges be allowed to choose their own trading rules? Recently, this issue has attracted considerable attention (see, for instance, Mahoney (1997), Kahan (1997) or Macey and O’Hara (1997)). One argument in favor of self regulation is that competition for listings should lead exchanges to choose trading rules that are socially desirable. For instance, Mahoney (1997) claims that:

“The necessity of attracting investors who have ample alternatives should lead exchanges to choose rules and listing standards that produce benefits to investors [...] Self-interested Stock Exchange members will produce rules that investors want for the same reasons that self-interested bakers produce the kind of bread that consumers want.” [Mahoney (1997), p1459]

Our measure of social welfare is the total surplus, \( \Lambda(q_1, q_2) \): the sum of total investors’ surplus, \( \Lambda^I(q_1, q_2) \), total entrepreneurs’ surplus, \( \Lambda^E(q_1, q_2) \), and total exchange surplus, \( \Pi_1 + \Pi_2 \).

In order to simplify the analysis, we assume that \( \gamma = 1 \) but our results do not depend on this assumption. When \( \gamma = 1 \), there are no transfers to third parties. Hence, exchange profits are transfers from the entrepreneurs and investors. Further, when computing social surplus, we restrict attention to parameter values so that in equilibrium all the firms go public (i.e. \( a \leq \frac{1}{3} \)) and both exchanges are active (\( \sigma > \sigma^c \)). We indicate why this is innocuous below. Computations yield the following closed form expression for total social welfare.

**Lemma 7**: Social Welfare when both exchanges choose a different trading technology is:

\[
\Lambda(q_1, q_2) = \left[ \frac{t^2 - \bar{t}^2}{8\sigma} + \frac{q_1(\bar{t}^3 - \bar{t}^3)}{48\sigma} - \frac{(t^3 - \bar{t}^3)\Delta q}{48\sigma} \right] N. \tag{17}
\]

Inspection of the previous equation, immediately yields the following proposition.

**Proposition 7**: Social welfare is maximized when both exchanges choose the high quality (low cost) technology, so that \( \Delta q = 0 \).
Distortions in this economy arise when an exchange optimally chooses to adopt a high trading cost (low quality), so that $\Delta q > 0$. Gains from trade between entrepreneurs and investors are then lower because the entrepreneurs who choose to list on the low quality (high cost) exchange issue fewer shares. Note, that if some firms do not go public ($a > 1/3$), the inefficiency is even stronger. An implication of this is that:

**Corollary 7**: If the dispersion of firm types in the economy is sufficiently large, $(\sigma > \sigma^e)$, then competition for listings between profit maximizing exchanges does not maximize social welfare.

Hence, competition between profit maximizing exchanges does not necessarily lead to socially efficient trading rules. Exchanges choose efficient trading rules, however if they choose their trading technology to maximize expected trading volume, under the constraint that they obtain a positive expected profit.\(^{22}\)

**Proposition 8**: If exchanges seek to maximize expected trading volume (instead of profit maximization), they both choose the low trading cost (high quality) technology and therefore social welfare is maximized.

Our two last results show that the efficiency of exchanges’ trading rules depends on their objective function and therefore on their governance structure. Intuitively, if security issuers owned or controlled exchanges, then they would be interested in maximizing the gains from trade (with investors) and would therefore choose a volume maximizing rule. This observation vindicates Amihud and Mendelson (1996)’s claim that the issuer should have a voice in decisions affecting the way its securities are traded. Recently many stock exchanges have been incorporated and thus became for-profit organizations.\(^{23}\) Our result suggests that a private, profit maximizing exchange

---

\(^{22}\)Huddart et al. (1999) consider the choice of disclosure requirements, in a Kyle (1985) model, by two Stock Exchanges competing for trading volume. The choice of a disclosure requirement in their model is similar to the choice of a trading technology in our model. Huddart et al. (1998) show that competition between exchanges results in a “race for the top” in the sense that both exchanges choose the highest possible disclosure requirement. Their analysis offers some support for the view that the choice of trading rules should be delegated to exchanges. We concur if exchanges are volume maximizers.

\(^{23}\)For instance the Australian, the Swedish and the Dutch Stock Exchanges. In the U.S., Nasdaq is considering issuing shares privately in the coming year.
with dispersed ownership may choose trading rules that are less socially desirable than a mutual, volume maximizing exchange.

To sum up, the results of this section show that self-regulation for a profit maximizing exchange does not necessarily yield a socially efficient trading organization, even in a competitive environment.

7 Extensions.

In this section, we present two extensions of our model of competition for listings. First, we show that our model can explain why exchanges voluntarily limit their market shares by setting minimum size requirements. Then, we consider entry in the exchange services market. We show that there is a natural limit on the number of exchanges, independent of the magnitude of entry costs.

7.1 Minimum Size Requirements

Listed firms must meet listing requirements chosen by exchanges. For instance, firms must have an aggregate market value and a net income in excess of a pre-specified threshold. As for trading technologies, competing exchanges do not in general choose identical listing requirements. But why would an exchange impose listing requirements that are more stringent than a competitor’s listing requirements? Or, why would an exchange deliberately choose to restrict its market share? Our purpose is to show that, as for trading technologies, exchanges have an incentive to choose different size requirements in order to soften competition. To quickly convey the intuition, we consider the following simple game that we call the listing requirements game. In this game, exchanges have the same trading cost, but each exchange can require a minimum size for listed firms.

To formalize the notion of a minimum size requirement, let \( t_{j}^{\min} \) be the minimum size requirement of Exchange \( j \), meaning that only firms with an expected payoff larger than \( t_{j}^{\min} \) can list on Exchange \( j \). Suppose that the two exchanges have the same trading technology \( (q_1 = q_2) \) and suppose that at date 1, they can choose between two minimum size requirements: soft in which case \( t_{j}^{\min} = \frac{1}{2} \) or tough in which case
Let $t^*_{max} = t_T > t$. Let Exchange 1 be the exchange with a tough listing policy. Suppose that Exchange 1 chooses its listing fee first.\footnote{When the two exchanges have \textit{different} minimum size requirements, there is no equilibrium in pure strategy, in the listing fee stage, if the exchanges determine their fees simultaneously. Thus, for simplicity, we assume that exchanges choose their fees in sequence. The results in this section do not qualitatively depend on the sequence of moves.} If the two exchanges have the same listing policy, one exchange is randomly selected to choose its listing fee first. Then,

\begin{proposition}
Consider the listing requirements game at the stage in which exchanges choose their listing fees (date 2). In equilibrium:

- If the two exchanges have different minimum size requirements, both exchanges earn strictly positive profits. Further, Exchange 1 and Exchange 2 charge listing fees so that they attract respectively firms with sizes in $[t_T, t]$ and $[t, t_T]$.

- If the two exchanges have the same minimum size requirement, both exchanges charge listing fees so that they obtain zero expected profits.

The intuition is as follows. If the two exchanges have the same minimum size requirement, they end up competing à la Bertrand for listings which leaves no room for profits. By contrast, if they have different minimum size requirements, Exchange 2 (with the low listing requirement) has monopoly power on all the entrepreneurs with an expected payoff in $[t, t_T)$. This exchange can attract \textit{all} the firms with a fee which is slightly lower than the fee of Exchange 1 but is better off charging the monopoly fee for the entrepreneurs that are not eligible to list on Exchange 1. Exchange 1 chooses a listing fee sufficiently small so that (i) it attracts all the entrepreneurs with type larger than $t_T$ and (ii) Exchange 2 has no incentive to attract some of these entrepreneurs by slightly undercutting. Exchange 1’s fee must therefore satisfy:

$$F^*_1 + \frac{\gamma N m}{8} = \Pi^*_2,$$

or

$$F^*_1 = \Pi^*_2 - \frac{\gamma N m}{8}, \tag{18}$$

where
where $\Pi^m_2$ is the expected profit of Exchange 2 when it chooses to charge the monopoly fee for firms in $[t_L, t_T]$. Using the reasoning of Proposition 5, we can show that the stage in which exchanges choose their minimum size requirement has two Nash equilibria in pure strategies. In both equilibria, exchanges choose different size requirements since this is a way to split the market and to guarantee strictly positive expected profits. Therefore:

Proposition 10: In equilibrium, the two exchanges choose different minimum size requirements.

We have considered the level of the tough minimum size requirement ($t_T$) as given. Note that the expected profit ($\Pi^m_2$) obtained by Exchange 2 must increase with this level. It follows (using Equation (18)) that $F^*_1$ also increases with $t_T$. Thus, in choosing the minimum size requirement, Exchange 1 faces a trade off between the level of its listing fee and the size of its market share (which decreases with the toughness of its minimum size requirement). For values of $t_T$ close to $t_L$, Exchange 1’s expected profit is strictly increasing with the level of its minimum size requirement. Consequently, if Exchange 1 could optimally pick this level, it would choose it in such a way that Exchange 2’s market share is not too small, i.e., in such a way that $t_T$ is not too close to $t_L$.

To sum up, we have shown in this section that minimum size requirements arise naturally as a way to soften price competition for listings and to sustain strictly positive expected profits.\(^{25}\)

7.2 Determinants of the Number of Exchanges.

So far, we have taken the number of firms in the economy as given. Now we consider the possibility of entry in the market for exchange services. Is there a natural bound on the number of exchanges and does the threat of entry drive exchanges to choose the low cost technology?

\(^{25}\)Of course, another possible explanation for minimum size requirements is that they act as a screening device, which help exchanges in their certification role (which is beyond the scope of the present paper).
To address these questions we extend the game. At date 0, a large pool of potential exchanges decide to enter the market for exchange services. On entry they incur a fixed entry cost, $\epsilon$. After the entry stage, the game proceeds as before.

**Proposition 11**: For all $\epsilon > 0$, when there are only two trading technologies:

1. If $\sigma \leq \sigma^c$, then at most one exchange is formed.

2. If $\sigma > \sigma^c$, then at most two exchanges are formed and choose different trading technologies ($q_1$ and $q_2$).

These results hold for all fixed costs, even arbitrarily small ones. They are driven by the fact that exchanges make positive profits if they differentiate. If they do not differentiate, they obtain zero profits and cannot recover the fixed cost of entry. If $\sigma \leq \sigma^c$, we have shown (Proposition 3) that only the exchange with the highest quality attracts listings. Exchanges, therefore pool on the highest quality which would drive profits to zero (Proposition 5). Thus, only one exchange enters.\footnote{Conditional on being alone in the market, this exchange would act as a monopolist.}

If $\sigma > \sigma^c$, the market is large enough for two exchanges to enter and earn positive profits (Proposition 4). They can therefore recover a (sufficiently low) fixed cost. Another exchange entering the market cannot differentiate (given that there are two trading technologies). Therefore, it would Bertrand compete with one of the existing exchanges, earn zero profits and would not be able to recover the cost of entry.

Thus, the number of different trading technologies puts a natural bound on the number of exchanges. The first part of the previous result also suggests that the number of exchanges is also limited by the size of the economy, $\sigma$. In order to show this more formally, we consider a large number ($N \geq 2$) of different trading technologies with qualities: $q_1 > q_2 > \ldots > q_N$. We define $\sigma^c(n) = (2m)\left[\frac{1-(\frac{2}{3})^{(n-1)}}{1+(\frac{2}{3})^{(n-1)}}\right]$

**Proposition 12**: For all $\epsilon > 0$ and for all $n \in [2, N]$, if $\sigma \leq \sigma^c(n)$ then at most $(n - 1)$ exchanges are formed. All the exchanges that are formed choose different trading technologies, namely $q_1, q_2, \ldots, q_{n-1}$.

Note that $\sigma^c(2) = \frac{2m}{3}$. Thus, when $\sigma \leq \frac{2m}{3}$, there cannot be more than one exchange operating profitably, even if the number of available trading technologies is
large. Furthermore, for $\sigma \in \left[ \frac{2m}{5}, \frac{10m}{13} \right]$, at most two exchanges can be profitably formed, even if more than two different trading technologies are available and if $\epsilon$ is very small. More generally, the result shows that a necessary condition for the formation of at least $n$ exchanges is $\sigma > \sigma(n)$. The intuition is the same as for Proposition 3 (which is a special case with $n = 1$ and $N = 2$). When $\sigma \leq \sigma(n)$, competition is so strong between the exchanges with the highest qualities that they attract all the listings. Thus, there is no room for another exchange to enter profitably, by providing the trading technology with quality (say) $q_{n+1}$. Exchanges which are formed choose different trading technologies because differentiation is the only way to obtain strictly positive profits and to recover entry costs (as in the previous proposition).

8 Conclusion.

This paper provides a model of competition for listings in which exchanges choose their trading rules and listing fees. We find that competition results in a variety of trading rules in equilibrium. This variety is a way for exchanges to soften competition. The general point that a choice of trading technology allows firms to differentiate themselves and so soften price competition can apply to other screening rules promulgated by exchanges (e.g. minimum size requirements). Interestingly, the number of exchanges that can be profitably formed depends both on the size of the economy and the number of different trading technologies. Further, the fact that trading rules can soften price competition implies that exchanges may take decisions with respect to trading rules that are not optimal for social welfare.

In our framework, we suggest that exchanges have some incentive to organize themselves so that the proportion of the trading costs that they recover are small ($\gamma$ in our model). Credible ways to do this are to have a large minimum tick size, or to organize themselves in such a way that third parties receive a benefit from trading volume. We also point that exchanges’ objectives and thereby decisions in our model would be different if they were controlled by issuers. These issues regarding

---

27This condition is not sufficient, however. For instance, if the entry cost is very large, a small number of exchanges will be formed, independently of the size of the economy ($\sigma$) or the diversity of trading technologies ($N$).
the governance of exchanges are interesting venues for future research.
9 Bibliography


Chemmanur, T. and Fulghieri, P. (1999): “Choosing an exchange to list equity”, mimeo, INSEAD.


### Table of Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>per share payoff</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>dispersion of firm’s payoffs</td>
</tr>
<tr>
<td>$m$</td>
<td>average per share payoff</td>
</tr>
<tr>
<td>$\delta$</td>
<td>entrepreneur’s discount factor</td>
</tr>
<tr>
<td>$N$</td>
<td>maximum number of shares</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>proportion of shares retained by entrepreneur</td>
</tr>
<tr>
<td>$s$</td>
<td>probability of liquidity shock for investor $s$</td>
</tr>
<tr>
<td>$c_L$</td>
<td>low trading cost</td>
</tr>
<tr>
<td>$c_H$</td>
<td>high trading cost</td>
</tr>
<tr>
<td>$q_L$</td>
<td>$\frac{1}{c_H}$, low quality</td>
</tr>
<tr>
<td>$q_H$</td>
<td>$\frac{1}{c_L}$, high quality</td>
</tr>
<tr>
<td>$F_j$</td>
<td>Listing Fee of Exchange $j$</td>
</tr>
<tr>
<td>$Vol(t, q_j)$</td>
<td>Expected trading volume of firm $t$ listed on Exchange $j$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>fraction of trading costs recovered by exchange</td>
</tr>
<tr>
<td>$p^{IPO}$</td>
<td>IPO price</td>
</tr>
<tr>
<td>$p^{SEC}$</td>
<td>price in the secondary market</td>
</tr>
<tr>
<td>$V(s, t, c_j)$</td>
<td>per share valuation of an investor of type $s$</td>
</tr>
<tr>
<td>$\Delta U(\alpha, t, c_j)$</td>
<td>utility benefit of going public on Exchange $j$</td>
</tr>
<tr>
<td>$\Delta q$</td>
<td>$q_H - q_L$, the quality difference</td>
</tr>
<tr>
<td>$\Delta F$</td>
<td>$F_1 - F_2$, the difference in fees</td>
</tr>
<tr>
<td>$a$</td>
<td>percentage difference in execution costs $\Delta q = \frac{a}{c_L}$</td>
</tr>
<tr>
<td>$\Lambda'$</td>
<td>total investors’ surplus</td>
</tr>
<tr>
<td>$\Lambda^E$</td>
<td>total entrepreneurs’ surplus</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>total social welfare</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Fixed entry cost</td>
</tr>
</tbody>
</table>
10 Appendix.

Proof of Lemma 1.

The result is immediate from the discussion that precedes the lemma. □

Proof of Lemma 2.

Given \( t \), an entrepreneur chooses the proportion he retains, \( \alpha \), to maximize

\[
\Delta U(\alpha, t, c_j) = (1 - \delta)N(1 - \alpha)t - (1 - \alpha)^2(Nc_j),
\]

which is concave in \( \alpha \). \( \alpha^* \) is obtained from the first order condition of the entrepreneur’s maximization problem (with \( q_j = 1/c_j \)). The IPO price is obtained from Equation (6) and the maximal utility benefit is obtained from Equation (8) (for \( \alpha = \alpha^* \)). □

Proof of Lemma 3.

The expected trade size for an investor with type \( s \) in a given firm is \( s(Nds) \). The marginal buyer in the IPO of a firm with type \( t \) has type \((1 - \alpha^*)\). It follows that the investors holding shares of this firm in their portfolios are uniformly distributed on \([0, (1 - \alpha^*)]\). The expected trading volume for a firm, conditional on being type \( t \), is therefore:

\[
\int_0^{(1-\alpha^*)} \frac{Nds}{(1 - \alpha^*)} = \frac{N(1 - \alpha^*)}{2} = \frac{(1 - \delta)Ntq_j}{4}.\]

Proof of Lemma 4.

Each part is obtained by differentiating \( \Delta U(\alpha^*(t, q_j), t) \) (Equation (10)) with respect to \( t \) and \( q \). □

Proof of Proposition 1.

Immediate from arguments in the text. □

Proof of Lemma 5.

Consider an entrepreneur of type \( t \) who goes public. According to Equation (14), this entrepreneur lists on Exchange 2 if:

\[
\frac{N[q_1 - q_2]t^2}{16} \leq F_1 - F_2.
\]

This inequality is binding for \( t^*(F_1, F_2) = 4\sqrt{\frac{\Delta F}{N\Delta q}} \). If \( t^* > t \), then firms smaller than \( t^* \) prefer to list on Exchange 2 (if they go public). If \( t^* < t \) then firms always
prefer to list on Exchange 1. □

Proof of Lemma 6

Suppose that 1) is not true so that there is a pair of listing fees \((F_1^*, F_2^*)\) such that only Exchange 2 attracts listings. Then \(F_1^* = F_1^{max}(F_2^*)\). But consider the deviation \(\tilde{F}_1 = F_1^{min}(F_2^*)\). Now Exchange 1 attracts all the listings and thus obtains a profit which is strictly larger than the profit obtained by Exchange 2 (since \(F_1^{min}(F_2^*) > F_2^*\)). In equilibrium, the profit of Exchange 2 is positive. Hence \(\tilde{F}_1\) is a profitable deviation and \((F_1^{max}(F_2^*), F_2^*)\) cannot be an equilibrium.

Suppose that 2) is not true so that some firms do not go public on Exchange 1. As the utility of going public is strictly positive, Exchange 2 can offer a positive listing fee that attracts unlisted firms. This is a profitable deviation of Exchange 2. Thus, 2) is true.

The last part of the lemma is a direct implication of the two first parts. □

Proof of Proposition 2.: 

We prove the proposition through a series of 3 lemmata. First, however, we establish a condition for a pure strategy equilibrium to exist.

Lemma 8: The condition \(\gamma \leq 2(\Delta q)t^*(F_1^*, F_2^*)\) is necessary for an equilibrium in pure strategies to exist.

Proof: Exchange 2’s expected profit is:

\[
\Pi_2(F_1^*, F_2) = \frac{(t^* - t)}{\sigma} \left[ F_2 + \frac{\gamma N}{16} (t + t^*) \right].
\]

Let \(g(F_2) = F_2 + \frac{\gamma N}{16} (t + t^*)\). This is the per listing profit for Exchange 2. We obtain:

\[
\frac{\partial \Pi_2(F_1^*, F_2^*)}{\partial F_2} = \frac{1}{\sigma} \left[ \frac{\partial t^*(F_1^*, F_2^*)}{\partial F_2} g(F_2) + \frac{\partial g(F_2)}{\partial F_2} (t^* - t) \right].
\]

If \(F_2^*\) is a best response for Exchange 2, the following condition holds:

\[
\frac{\partial \Pi_2(F_1^*, F_2^*)}{\partial F_2} \geq 0.
\]
If this holds as an equality then $F_2^*$ is an interior solution to Exchange 2’s maximization problem and $F_2^* = F_2^{int}$. If the inequality is strict then $F_2^*$ is a corner solution to Exchange 2’s maximization problem and $F_2^* = F_2^{max}$. These are the only two possibilities (see Lemma 6).

When Exchange 2 increases its fee, its market share decreases thus the first term in bracket in Equation (19) is negative. Thus, a necessary condition for the previous inequality to be satisfied is $\frac{\partial g(F_2)}{\partial F_2} \geq 0$. That is, the per listing profit must increase with the listing fee (at least at the equilibrium listing fee for Exchange 2). When $\gamma > 0$, this may not be the case since the decrease in the number of listings translates into a decrease in the revenue obtained from trading fees. In fact:

$$\frac{\partial g(F_2)}{\partial F_2} = 1 + \gamma N \frac{\partial t^*(F_1^*, F_2)}{\partial F_2}.$$  

From the definition of $t^*$ (See Equation (15)), we get:

$$\frac{\partial t^*(F_1, F_2)}{\partial F_1} = -\frac{\partial t^*(F_1, F_2)}{\partial F_2} = \frac{2}{\sqrt{N \Delta q(\Delta F)}}. \quad (20)$$

It follows that:

$$\frac{\partial g(F_2)}{\partial F_2} = 1 - \frac{\gamma N}{8\sqrt{(N \Delta q(\Delta F))}},$$

and using Equation (15) again, this equation is rewritten as:

$$\frac{\partial g(F_2)}{\partial F_2} = 1 - \frac{\gamma}{2(\Delta q)t^*(F_1^*, F_2)}. \quad (21)$$

It follows that $\frac{\partial g(F_2)}{\partial F_2} \geq 0$ iff $2(\Delta q)t^*(F_1^*, F_2) \geq \gamma$. This is the case if $2(\Delta q)t \geq \gamma$ since $t^* \geq t$. □

**Lemma 9**: In any equilibrium in which Exchange 2 does not attract any listings then Exchange 1’s listing fee satisfies $F_1^* \leq \left[\frac{(\Delta q)^2}{16} - \frac{\Delta q}{8}\right] N$.

**Proof**: For $t^* > t$, Exchange 2’s expected profit when it attracts all the firms in $[t, t^*]$ is:

$$\Pi_2(F_1^*, F_2) = \frac{(t^* - t)}{\sigma} [F_2 + \frac{\gamma N}{16}(t + t^*)],$$
and since \( t^* > \hat{t} \):

\[
\Pi_2(F_1^*, F_2) > \frac{(t^* - \hat{t})}{\sigma} [F_2 + \gamma N_16(2\hat{t})].
\]

Thus when \( F_2 > -\frac{N_\gamma t}{8} \), the R.H.S is strictly positive. If \( F_2^{\text{max}}(F_1^*) > -\frac{N_\gamma t}{8} \), Exchange 2 can obtain a strictly positive profit by choosing a fee in \((-\frac{N_\gamma t}{8}, F_2^{\text{max}}(F_1^*))\). Hence \( F_2^{\text{max}}(F_1^*) \leq -\frac{N_\gamma t}{8} \) is necessary for Exchange 2 to have no other profitable choice than \( F_2^{\text{max}}(F_1^*) \). Recall, that \( F_2^{\text{max}}(F_1^*) = F_1^* - \frac{\Delta q t^2}{16} \). Therefore the condition \( F_2^{\text{max}}(F_1^*) \leq -\frac{N_\gamma t}{8} \) implies \( F_1^* \leq \left[ \frac{(\Delta q t^2}{16} - \frac{\gamma t}{8} \right] N \). □

**Lemma 10**: Recall that \( \sigma^c \equiv \frac{2m}{5} \). If \( \sigma > \sigma^c \) then both exchanges attract listings in equilibrium.

**Proof**: Using the definition of Exchange 1’s expected profit:

\[
\frac{\partial \Pi_1}{\partial F_1}(F_1, F_2) = \left[ (\bar{t} - t^*) - \frac{\partial t^*}{\partial F_1} F_1 - \frac{N_\gamma t^*}{8} \frac{\partial t^*}{\partial F_1} \right], \quad \text{for} \quad F_1 \geq F_1^{\text{min}}(F_2) \tag{22}
\]

and from the definition of \( t^* \) (See Equation (15)), we get:

\[
\frac{\partial t^*}{\partial F_1}(F_1, F_2) = -\frac{\partial t^*}{\partial F_2} = \frac{2}{\sqrt{N\Delta q(F_1 - F_2)}}. \tag{23}
\]

Note that \( F_1^{\text{min}}(F_2) - F_2 = \frac{N(\Delta q t^2)}{16} \). Using this and the fact that \( t^*(F_1^{\text{min}}(F_2), F_2) = \hat{t} \), we obtain:

\[
\frac{\partial \Pi_1}{\partial F_1}(F_1^{\text{min}}(F_2), F_2) = \frac{[\sigma - \frac{8F_1^{\text{min}}(F_2)}{N(\Delta q t^2)} - \frac{\gamma t}{\Delta q}]}{\sigma}.
\]

Since \( F_1^{\text{min}}(F_2) = F_2 + \frac{N(\Delta q t^2)}{16} \), this yields:

\[
\frac{\partial \Pi_1}{\partial F_1}(F_1^{\text{min}}(F_2), F_2) = \frac{\left[ \frac{5(\sigma N^2)}{4} - \frac{8F_2}{N(\Delta q t^2)} - \frac{\gamma t}{\Delta q} \right]}{\sigma}. \tag{24}
\]

Consider an equilibrium in which Exchange 2 attracts no listings. In this case the pair of equilibrium listing fees is \((F_1^*, F_2^*)\) with \( F_1^* = F_1^{\text{min}}(F_2^*) \) and \( F_2^* = F_2^{\text{max}}(F_1^*) \). Furthermore, from Lemma 9, in this case, \( F_1^* \leq \left[ \frac{(\Delta q t^2}{16} - \frac{\gamma t}{8} \right] N \). This yields \( F_2^{\text{max}}(F_1^*) \leq -\frac{N_\gamma t}{8} \). Using this fact and Equation (24), we obtain:
\[
\frac{\partial \Pi_1(F_1^{\text{min}}(F_2^*), F_2^{\text{max}}(F_1^*))}{\partial F_1^*} \geq \frac{5(\sigma - \frac{2m}{5})}{4\sigma}.
\]

If \( \sigma > \sigma^c \), the R.H.S is strictly positive. This means that Exchange 1 is better off raising its listing fee and \( F_1^{\text{min}}(F_2^*) \) cannot be a best response. This implies that \( (F_1^{\text{min}}, F_2^{\text{max}}) \) cannot be an equilibrium when \( \sigma > \sigma^c \). \(\square\)

**Proof of Proposition 3.**

We first show that \( \sigma > \sigma^c \) is a necessary condition for Exchange 1 to choose a cream skimming policy. In this case, the optimal listing fee, \( F_1^* \) for Exchange 1 is an interior solution to its maximization problem and therefore solves:

\[
\frac{\partial \Pi_1(F_1^*, F_2^*)}{\partial F_1^*} = 0,
\]

or

\[
(\bar{t} - t^*) - \frac{\partial t^*}{\partial F_1} F_1^* - \frac{N\gamma t^*}{8} \frac{\partial t^*}{\partial F_1} = 0.
\]

This equation is equivalent to:

\[
(\bar{t} - t^*) - \frac{\partial t^*}{\partial F_1} (F_1^* - F_2^*) - \frac{\partial t^*}{\partial F_1} (F_2^* + \frac{N\gamma t^*}{8}) = 0.
\]

Now using Equation (23), we obtain:

\[
\frac{\partial t^*}{\partial F_1} (F_1^* - F_2^*) = \frac{t^*}{2}
\]

Furthermore, since \( t^* > \bar{t} \), note that:

\[
F_2^* + \frac{N\gamma t^*}{8} > F_2^* + \frac{N\gamma (\bar{t} + t^*)}{16}
\]

The R.H.S of this equation is the per listing profit of Exchange 2 and must positive. Therefore, using Equation (27), we conclude that:

\[
(\bar{t} - t^*) + \frac{t^*}{2} > 0,
\]

which means that \( \bar{t} > \frac{3}{2} t^* \). Since Exchange 1 is assumed to choose the cream skimming policy, \( t^* > \bar{t} \). Therefore the previous inequality requires \( \bar{t} > \frac{3}{2} \bar{t} \), which means \( \sigma > \sigma^c \).

It follows that when \( \sigma \leq \sigma^c \), there is no equilibrium in which Exchange 2 attracts some
listings. Thus the equilibrium must be such that only Exchange 1 attracts listings.

It is easy to show that

$$F^*_1 = \left[ \frac{(\Delta q)t^2}{16} - \frac{\Delta q}{8} \right] N \quad \text{and} \quad F^*_2 = -\frac{N\Delta q}{8}$$

is an equilibrium in this case. □

**Proof of Proposition 4.**

Let $F^*_2$ be the fee of Exchange 2 in equilibrium. By choosing a fee equal to $F^*_1 = (\Delta q)t \gamma_2^*$, Exchange 1 attracts all the listings and thus obtains a profit which is strictly larger than the profit obtained by Exchange 2 (since $F^*_1 > F^*_2$). This means that $\Pi_1(F^*_1, F^*_2) > \Pi_2(F^*_1, F^*_2)$. Since $\Pi_1(F^*_1, F^*_2) \geq \Pi_1(F^*_1, F^*_2)$, we deduce that $\Pi_1(F^*_1, F^*_2) > \Pi_2(F^*_1, F^*_2)$. This proves that $\Pi_H > \Pi_L$.

To prove that $\Pi_L > 0$, consider $(F^*_1, F^*_2)$ an equilibrium in which the two exchanges attract listings. This means that $t^* > \frac{\Delta q}{2}$ and $F^*_2$ is an interior solution. This implies that:

$$\frac{\partial \Pi_2(F^*_1, F^*_2)}{\partial F^*_2} = 0.$$  \quad (28)

Recall that (see Lemma 8 in the Proof of Proposition 2):

$$\frac{\partial \Pi_2(F^*_1, F^*_2)}{\partial F^*_2} = \frac{1}{\sigma} \left[ \frac{\partial t^*(F^*_1, F^*_2)}{\partial F^*_2} g(F^*_2) + \frac{\partial g(F^*_2)}{\partial F^*_2} (t^* - t) \right]$$

Now suppose that in equilibrium, Exchange 2 attracts some listings but obtains a zero expected profit. This means that its per listing profit is zero, that is $g(F^*_2) = 0$. But then Equation (28) implies:

$$\frac{\partial g(F^*_2)}{\partial F^*_2} = 0.$$  \quad (29)

Using Equation (21), this imposes $t^*(F^*_1, F^*_2) = \frac{2}{\gamma_2}$, which is not true, generically. In particular, it cannot be satisfied if $\gamma < 2(\Delta q)/\Delta q$. □

**Proof of Proposition 5.** Immediate from the arguments in the text. □

**Proof of Corollary 1.:**

Let $I$ be an indicator variable that is equal to 1 conditional on a firm listing on Exchange 1. The proportion of shares sold to investors in the IPO of a firm with type $t$ that lists on Exchange $j$ is $(1 - \alpha^*(t, q_j)) = \frac{q_j}{4}$. Thus the expected proportion of original shares sold to the public conditional on the IPO taking place on Exchange 1 is:

$$E((1 - \alpha^*(t, q_1)) \mid I = 1) = \frac{q_1}{4} E(t \mid t > t^*).$$
In the same way:

\[ E((1 - \alpha^* (t, q_2)) \mid I = 0) = \frac{q_2}{4} E(t \mid t \leq t^*). \]

As \( q_1 > q_2 \), we obtain \( E((1 - \alpha^* (t, q_1)) \mid I = 1) > E((1 - \alpha^* (t, q_2)) \mid I = 0). \)

**Proof of Corollary 2.**

We use the indicator function defined in the previous corollary. The post IPO market value of a firm \( t \) that lists on Exchange \( j \) is \( Np^{IPO}(\alpha^*(t, q_j), t, c_j) = \frac{3Nt^2}{4}. \)

Consequently,

\[ E(Np^{IPO}(\alpha^*(t, q_1), t, c_1) \mid I = 1) > E(Np^{IPO}(\alpha^*(t, q_2), t, c_2) \mid I = 0), \]

which proves the last part of the proposition. The IPO proceeds of a firm with type \( t \) that lists on Exchange \( j \) are equal to:

\[ N(1 - \alpha^*(t, q_j))p^{IPO}(\alpha^*, t, c_j) = \frac{3Nt^2q_j}{16}. \]

Consequently,

\[ E(N(1 - \alpha^*(t, q_1))p^{IPO}(\alpha^*, t, c_1) \mid I = 1) > E(N(1 - \alpha^*(t, q_2))p^{IPO}(\alpha^*, t, c_2) \mid I = 1), \]

which proves the second part of the corollary.\( \square \)

**Proof of Corollary 3.**

The demand for shares in the IPO is given in Lemma 1. Corollary 3 follows.\( \square \)

**Proof of Proposition 6.**

If the two exchanges attract listings then \( t^* > \bar{t} \). This implies that listing fees are interior solutions to the Exchange maximization problems. Thus, the listing fees solve the first order conditions for exchanges’ maximization problems. Let \( F_1^{int} \) and \( F_2^{int} \) be these solutions, respectively for Exchange 1 and 2. Consider the case in which these fees are such that all the firms go public. The first order condition for Exchange 1 is:

\[ (\bar{t} - t^*) - \frac{\partial t^*}{\partial F_1} F_1^{int} - \frac{N\gamma t^*}{8} \frac{\partial t^*}{\partial F_1} = 0. \]  

(29)

For Exchange 2:

\[ (t^* - \bar{t}) + \frac{\partial t^*}{\partial F_2} F_2^{int} + \frac{N\gamma t^*}{8} \frac{\partial t^*}{\partial F_2} = 0. \]  

(30)
Subtracting Equation (30) from Equation (29), we get that:

\[
(t + \bar{t} - 2t^*) = \left( \frac{\partial t^*}{\partial F_1} \right) (F_1^{int} - F_2^{int}).
\]  

(31)

Now,

\[
\frac{\partial t^*}{\partial F_1} = -\frac{\partial t^*}{\partial F_2} = \frac{2}{\sqrt{N\Delta q(F_1 - F_2)}},
\]  

(32)

Using Equation (32) at the point \((F_1^{int}, F_2^{int})\) and the definition of \(t^*\), Equation (31) yields:

\[
(t + \bar{t} - 2t^*) = \frac{t^*}{2},
\]

which implies \(t^* = \frac{4m}{5}\). Using this fact and Equation (15), we obtain that \((F_1^{int}, F_2^{int})\) must be such that:

\[
\sqrt{(F_1^{int} - F_2^{int})} = \frac{m\sqrt{N\Delta q}}{5}.
\]  

(33)

It follows from Equation (32):

\[
\frac{\partial t^*}{\partial F_1} = \frac{10}{N(\Delta q)m}
\]

We can then easily solve Equations (29) and (30) for \(F_1^{int}\) and \(F_2^{int}\). These solutions are as announced in the proposition. Note that since \(t^* = \frac{4m}{5}\), it is possible to rewrite \(F^{int}(= F_2^*)\) as:

\[
F_2^{int} = [(t^* - \bar{t}) \frac{m}{10} (\Delta q) - \frac{\gamma m}{10}]N.
\]  

(34)

Recall that \(\Delta U(\alpha^*(t, q_2), \bar{t}) = \frac{q_2N\alpha^2}{16}\). Hence all the firms go public if \(F_2^{int} \leq \frac{q_2N\alpha^2}{16}\).

Using Equation (34), it is readily shown that a sufficient condition is \(\frac{\Delta q}{q_2} < \frac{t^2}{2(t^* - \bar{t})} \bar{t}\).

The R.H.S of this inequality is lower than 1/2. Furthermore \(\frac{\Delta q}{q_2} = \frac{a}{1-a}\). It follows that the sufficient condition is satisfied if \(\frac{a}{1-a} < \frac{1}{2}\) or \(a < \frac{1}{3}\).

Now, we need to establish that \(F_1^{int}\) and \(F_2^{int}\) dominate any other listing fees that the two exchanges could choose. To do so, we establish that \(F_1^{int}\) and \(F_2^{int}\) are local maxima. Then, we show that these local maxima are indeed global maxima.

Consider Exchange 1 first. We obtain:
\[
\frac{\partial^2 \Pi_1(F_1, F_2)}{\partial^2 F_1} = - \left[ 2 \frac{\partial t^*}{\partial F_1} - \frac{\gamma N}{8} \left( \frac{\partial t^*}{\partial F_1} \right)^2 - \frac{\partial^2 t^*}{\partial^2 F_1} \left( \frac{\gamma t^* N}{8} + F_1 \right) \right] \sigma^{-1}, \quad \forall F_1 \in [F_1^{\text{min}}, F_1^{\text{max}}],
\]

with \( \frac{\partial^2 F}{\partial F_1} = -(F_1 - F_2)^{-\frac{3}{2}}/(N \Delta q)^{\frac{3}{2}} \). Using Equation (32), algebra yields:

\[
\frac{\partial^2 \Pi_1(F_1, F_2)}{\partial^2 F_1} = -[3F_1 - 4F_2] \left( \frac{\sqrt{\Delta F}}{2N \Delta q \sigma} \right).
\]

(35)

It follows that for \( F_1 \geq F_1^{\min}(F_2) \):

\[
\frac{\partial^2 \Pi_1(F_1, F_2)}{\partial^2 F_1} \leq -[3F_1^{\min} - 4F_2] \left( \frac{\sqrt{\Delta F}}{2N \Delta q \sigma} \right).
\]

(36)

Using the characterization of \( F_1^{\text{int}} \) and \( F_2^{\text{int}} \), we obtain:

\[
\frac{\partial^2 \Pi_1(F_1^{\text{int}}, F_2^{\text{int}})}{\partial^2 F_1} < 0.
\]

Hence, \( F_1^{\text{int}} \) is a local maximum. For a given \( F_2^{\text{int}} \), Equation (29) is quadratic in \( F_1 \). Thus, this equation has another solution. But this solution is a local minimum since \( F_1^{\text{int}} \) is a local maximum.\(^{28}\) We still have to check that there is no corner solution, that is: \( \Pi_1(F_1^{\text{int}}, F_2^{\text{int}}) > \Pi_1(F_1^{\text{min}}, F_2^{\text{int}}) \) and that \( \Pi_1(F_1^{\text{int}}, F_2^{\text{int}}) > \Pi_1(F_1^{\text{max}}, F_2^{\text{int}}) \). Tedious computations yield:

\[
\Pi_1(F_1^{\text{int}}, F_2^{\text{int}}) - \Pi_1(F_1^{\text{min}}, F_2^{\text{int}}) = \left[ \left( \frac{\sigma}{2} - \frac{m}{5} \right) \right] \left[ \left( \frac{N \sigma \Delta q + \gamma}{16} \right) \left( 1 + \frac{4m}{5} \right) - F_1^{\text{int}} \right].
\]

The first term in bracket is positive since \( \sigma \geq \sigma^c \). The second term in bracket can be shown to be positive as well because \( m \geq \sigma \). Hence, \( \Pi_1(F_1^{\text{int}}, F_2^{\text{int}}) - \Pi_1(F_1^{\text{min}}, F_2^{\text{int}}) > 0 \). Now \( \Pi_1(F_1^{\text{min}}, F_2^{\text{int}}) > \Pi_2(F_1^{\text{int}}, F_2^{\text{int}}) \geq 0 \). The first inequality is straightforward and the second is proved below when we show that \( F_2^{\text{int}} \) is a global maximum. This implies \( \Pi_1(F_1^{\text{int}}, F_2^{\text{int}}) > 0 \). With a listing fee equal to \( F_1^{\text{max}} \), Exchange 1’s market share is zero which implies \( \Pi_1(F_1^{\text{max}}, F_2^{\text{int}}) = 0 \). Thus \( F_1^{\text{int}} \) is preferred to \( F_1^{\text{max}} \). This proves that \( F_1^{\text{int}} \) is a global maximum or \( F_1^{\text{int}} = F_1^{\max}(F_2^{\text{int}}) \).

\(^{28}\)If \( \Pi_1(., F_2^{\text{int}}) \) had two local maxima in \( (F_1^{\text{min}}, F_1^{\text{max}}) \) then it would necessary have a local minimum in this interval as well. But this would imply that the first order condition has 3 solutions, which is impossible since this condition is a quadratic equation.
For Exchange 2, we obtain:

\[
\frac{\partial^2 \Pi_2(F_1, F_2)}{\partial^2 F_2} = -[4F_1 - 3F_2](\frac{\sqrt{\Delta F}}{\sqrt{N\Delta q\sigma}}).
\] (37)

Since \(F^\text{int}_1 > F^\text{int}_2\), we obtain \(\frac{\partial^2 \Pi_2(F^\text{int}_1, F^\text{int}_2)}{\partial^2 F_2} < 0\), which proves that \(F^\text{int}_2\) is a local maximum. Following the same reasoning as for Exchange 1, we can show that this local maximum is unique. It remains to show that \(\Pi_2(F^\text{int}_1, F^\text{int}_2) > \Pi_2(F^\text{int}_1, F^\text{max}_2)\) and \(\Pi_2(F^\text{int}_1, F^\text{int}_2) > \Pi_2(F^\text{int}_1, F^\text{max}_2)\). With \(F^\text{max}_2\), Exchange 2 attracts no listings and gets a zero profit. We have \(\Pi_2(F^\text{int}_1, F^\text{int}_2) = (\frac{t^* - t}{\sigma})[F^\text{int}_2 + \frac{\gamma N}{16}(l + t^*)]\). Thus, for Exchange 2 to get a positive expected profit with a listing fee equal to \(F^\text{int}_2\), we need:

\[
F^\text{int}_2 \geq -\frac{\gamma N(l + t^*)}{16}.
\]

Using Equation (34) and the fact that \(t^* = \frac{4m}{5}\), we find that this inequality is satisfied iff \(\gamma < 2(\Delta q)t^*\), which is true (recall that we assume \(\gamma < 2(\Delta q)t\)). Hence, \(\Pi_2(F^\text{int}_1, F^\text{int}_2) > \Pi_2(F^\text{int}_1, F^\text{max}_2)\). Under the same condition, it is also possible to show that \(\Pi_2(F^\text{int}_1, F^\text{min}_2) < 0\). It follows that \(\Pi_2(F_1^\text{int}, F_2^\text{int}) > \Pi_2(F_1^\text{int}, F_2^\text{min})\). Consequently \(F^\text{int}_2\) is a global maximum or \(F^\text{int}_2 = F^\gamma_2(F^\text{int}_1)\).

**Proof of Corollary 4.**

Listing fees in Proposition 6 are linear functions of the quality differential, \((\Delta q)\). The slopes of these functions are \(b_1(\sigma)\) and \(b'_1(\sigma)\). They are positive if \(\sigma \geq \sigma_c\).

**Proof of Corollary 5.**

Immediate using the closed form solutions of the listing fees given in Proposition 6.

**Proof of Corollary 6.**

Listing fees in Proposition 6 are linear functions of \(\gamma\). The slopes of these functions are negative.

**Proof of Lemma 7.**

If exchanges have different trading technologies, then, the total surplus to entrepreneurs, \(\Lambda^E(q_1, q_2)\) is just the excess of their utility from going public over the listing fees that they have to pay:

\[
\Lambda^E(q_1, q_2) = \int_{t^*}^{t^*} \frac{\Delta U(t, q_2) - F_2}{\sigma} dt + \int_{t^*}^{t} \frac{\Delta U(t, q_1) - F_1}{\sigma} dt
\]
An investor of type $s$ who buys into an IPO gets a benefit, per share, of the difference between her valuation and the IPO price, or $[t - sc] - [t - (1 - \alpha^*)c]$. So, the aggregate expected surplus captured by investors in all IPOs is just:

$$\Lambda^I(q_1, q_2) = \int_t^{t^*} \left[ \int_0^{(1-\alpha^*)} \left( (1 - \alpha^*(t, q_2)) - (1 - \alpha^*(t, q_1)) - s \right) c_2 \frac{N ds}{1-\alpha^*} \right] \frac{dt}{\sigma} + \int_t^{t^*} \left[ \int_0^{(1-\alpha^*)} \left( (1 - \alpha^*(t, q_1)) - s \right) c_1 \frac{N ds}{1-\alpha^*} \right] \frac{dt}{\sigma},$$

where expectations are taken over investors who buy into the IPOs and over firm types. Recall that $(1 - \alpha^*(t, q_j)) = \frac{t q_j}{4}$ and that the expected trading volume of a firm with type $t$ listed on Exchange $j$ is: $Vol(t, q_j) = \frac{N t q_j}{\sigma}$. It follows that:

$$\int_0^{(1-\alpha^*)} \left[ \frac{(1 - \alpha^*) - s}{(1 - \alpha^*)} N c_j ds \right] = \frac{N t}{4} - c_j Vol(t, q_j).$$

Therefore, the total surplus to investors is:

$$\Lambda^I(q_1, q_2) = \int_t^{t^*} \left[ \frac{N_t - c_2 Vol(t, q_2)}{\sigma} \right] dt + \int_t^{t^*} \left[ \frac{N_t - c_1 Vol(t, q_1)}{\sigma} \right] dt.$$

The total expected surplus is:

$$\Lambda = \Lambda^I(q_1, q_2) + \Lambda^E(q_1, q_2) + \Pi_1 + \Pi_2$$

Since $\gamma = 1$, listing fees and trading costs are transfers from investors and firms to the exchange. Therefore,

$$\Lambda = \int_t^{t^*} \frac{N_t}{4\sigma} dt + \int_t^{t^*} \frac{N_t}{4\sigma} dt + \int_t^{t^*} \frac{\Delta U(t, q_2)}{\sigma} dt + \int_t^{t^*} \frac{\Delta U(t, q_1)}{\sigma} dt.$$

Finally using the fact that $\Delta U(\alpha^*(t, q_j), t) = \frac{N t^2 q_j}{16}$, we obtain:

$$\Lambda(q_1, q_2) = \left[ \frac{t^2 - t^2}{8\sigma} + \frac{q_1 (t^3 - t^3)}{48\sigma} - \frac{(t^3 - t^3) \Delta q}{48\sigma} \right] N, \Box$$

**Proof of Proposition 7**

The total expected surplus, $\Lambda$, decreases with $\Delta q$. Hence welfare is maximized for $\Delta q = 0. \Box$

**Proof of Corollary 7.**
When $\sigma > \sigma^c$, exchanges optimally choose different trading technologies. Thus $\Delta q > 0$ and welfare is not maximized.\(\Box\)

**Proof of Proposition 8.**

Suppose first that the two exchanges have different trading technologies and assume that Exchange 2’s listing fee is $F_2^\ast$. Let $E^T Vol(q_j)$ be the total expected trading volume in Exchange $j$. Consider the objective function of Exchange 1:

$$Max_{F_1} E^T Vol(q_1) = \int_{t_c}^{\bar{t}} \frac{Ntq_1}{8\sigma} dt = \frac{Nq_1(\bar{t}^2 - t_c^2)}{16\sigma},$$

s.t. $\Pi_1(F_1, F_2^\ast) \geq 0,$

with $t_c = \max\{t, 4\sqrt{\frac{F_1 - F_2^\ast}{N\Delta q}}\}$. Clearly, Exchange 1 maximizes its expected trading volume by choosing its fee so that $t_c = t$. Now consider the following listing policy for Exchange 1 and Exchange 2: $F_1^\ast = N[\frac{\bar{t}^2\Delta q}{16} - \frac{2m}{8}]$ and $F_2^\ast = -\frac{N\gamma m}{8}$. Note that $t^*(F_1^\ast, F_2^\ast) = \bar{t}$. The smallest fee that Exchange 2 can charge without losing money is $F_2^\ast$ given the fee of Exchange 1. Thus $F_2^\ast$ is a best response for Exchange 2. Furthermore $\Pi_1(F_1^\ast, F_2^\ast)$ is strictly positive. Therefore $F_1^\ast$ is a solution of the previous maximization program and $(F_1^\ast, F_2^\ast)$ is an equilibrium. The expected trading volume in the two exchanges in this case are respectively:

$$E^T Vol(q_1) = \frac{Nq_1(\bar{t}^2 - t_c^2)}{16\sigma},$$

$$E^T Vol(q_2) = 0.$$

If the two exchanges choose the same trading technology, they compete à la Bertrand. The equilibrium outcome is identical to the Nash equilibrium described in Proposition 1 and each exchange attracts half of the listings. In this case the expected trading volume of each exchange is:

$$E^T Vol(q) = \frac{Nq(\bar{t}^2 - t_c^2)}{32\sigma},$$

where $q$ is the quality of the trading technology chosen by both exchanges.

Now consider the stage (date 1) in which exchanges choose their trading technology with a view at maximizing their expected trading volume. The following table gives the expected trading volume of each exchange according to their trading technology.
choice, where $q_2$ denotes the high trading cost trading technology and $q_1$ the low trading cost technology.

<table>
<thead>
<tr>
<th>$q_2$</th>
<th>$q_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(Nq_2(t^2 - t^2), \frac{Nq_2(t^2 - t^2)}{32\sigma})$</td>
<td>$(0, \frac{Nq_1(t^2 - t^2)}{16\sigma})$</td>
</tr>
<tr>
<td>$(\frac{Nq_1(t^2 - t^2)}{16\sigma}, 0)$</td>
<td>$(\frac{Nq_1(t^2 - t^2)}{32\sigma}, \frac{Nq_1(t^2 - t^2)}{32\sigma})$</td>
</tr>
</tbody>
</table>

Table 2

As $q_1 > q_2$, it is a strictly dominant strategy for an exchange to choose the low trading cost technology. □

**Proof of Proposition 9.**

First suppose that the two exchanges have different size requirements. We first derive the optimal reaction of Exchange 2 to the listing fee $F_1^*$ chosen by Exchange 1. Let $F_m(t, t_T)$ be the listing fee that is charged by an exchange which is a monopolist over a population of entrepreneurs with types in $[t, t_T]$. Suppose that the listing fee of Exchange 1 is lower than $F_m(t, t_T)$. Exchange 2 can choose between two strategies. First it can undercut slightly Exchange 1. In this way it captures all the listings since the two exchanges have the same trading technology. It obtains a profit equal to:

$$F_1 + \frac{\gamma N(\bar{t} + t)}{16}.$$

Alternatively Exchange 2 can exploit its monopoly power on the entrepreneurs with a type in $[t, t_T]$ since these entrepreneurs are not eligible to list on Exchange 1. In this case, it charges a fee $F_2^* = F_m(t, t_T)$ and gets an expected profit:

$$\Pi_2^m = \left(\frac{t_T - t_m}{\sigma}\right) \left(F_m(t, t_T) + \frac{\gamma N}{16}(t_T + t_m)\right)$$

where $t_m \in [t, t_T]$ is the type of the entrepreneur who is just indifferent between listing on Exchange 2 or not going public. In order to be active Exchange 1 must choose its listing fee so that Exchange 2 is better off charging a fee equal to $F_m(t, t_T)$. Thus, Exchange 1 chooses a fee equal to:

$$F_1^* = \Pi_2^m - \frac{\gamma N(\bar{t} + \bar{t})}{16}.$$
Clearly Exchange 2 gets a strictly positive expected profit since \( F_m(\underline{t},t_T) \geq \frac{N \epsilon^2 q_2}{16} \).

Note that \( F_1^* < F_m(\underline{t},t_T) \) as it was assumed initially since:

\[
F_1^* - F_m(\underline{t},t_T) = \frac{(t_T - t_m)F_m - \sigma F_m}{\sigma} + \frac{\gamma N[(t_2^T - t_m^2) - (\bar{t}^2 - t_m^2)]}{16\sigma} < 0
\]

Exchange 1’s expected profit is

\[
\Pi_1 = \frac{(\bar{t} - t_T)}{\sigma} \left( F_1^* + \frac{\gamma N}{16} (\bar{t} + t_T) \right)
\]

It is strictly positive iff:

\[
F_1^* > -\frac{\gamma N (\bar{t} + t_T)}{16}
\]

This is equivalent to:

\[
\Pi_2^m > -\frac{\gamma N}{16} (t_T - \underline{t}),
\]

which is satisfied since \( \Pi_2^m > 0 \). This proves Part 1 of the lemma. If the two exchanges have the same minimum size requirement, the exchange with the lowest listing fee get all the listings. It is then direct that fees are chosen such that it cannot be optimal for the exchange that moves in second to undercut the first exchange. The only fee with this property is such that both exchanges get zero expected profits, which proves the second part. \( \square \)

**Proof of Proposition 10.**

Immediate from the arguments in the text. \( \square \)

**Proof of Proposition 11.**

From Proposition 1, if two exchanges enter and choose the same trading technology then profits are zero. Hence for \( \epsilon > 0 \), entry of two exchanges can be optimal only if subsequently they choose different trading technologies and attract listings.

Suppose \( \sigma \leq \sigma^c \). If two exchanges enter and differentiate, only one attracts all the listings (Proposition 3). Therefore only one exchange can be formed. For \( \epsilon > 0 \), sufficiently low, entry for a single exchange is profitable since it acts as a monopolist.

Suppose \( \sigma > \sigma^c \). From Proposition 4 if two exchanges enter and differentiate, they capture strictly positive profits. Hence, there exists an \( \epsilon > 0 \) sufficiently low, so that entry is profitable. Suppose that a third exchange enters. If it chooses a trading
technology with \( q_j, j = 1, 2 \), the two exchanges with this quality Bertrand compete and make zero profits. Thus a third exchange cannot be formed profitably. \( \square \)

**Proof of Proposition 12.**

In this proof Exchange \( j \) is the exchange with quality level \( j \). Suppose that there exists an equilibrium in which \( n \geq 2 \) exchanges are formed. Two exchanges with the same trading technology Bertrand compete and obtain zero profits. Thus, the exchanges which enter later must choose different trading technologies in order to recover the entry cost. It is immediate that these exchanges must choose the trading technologies with the \( n \) first qualities: \( \{q_1, q_2, \ldots, q_n\} \). Otherwise an additional exchange could be formed profitably by choosing the trading technology which is not provided in the set \( \{q_1, q_2, \ldots, q_n\} \).

Now, we prove that a necessary condition for the \( n \) exchanges which enter to obtain strictly positive expected profits, is that \( \sigma > \sigma(n) \). Let \( F^* = (F^*_1, \ldots, F^*_n) \) be the vector of listing fees chosen by Exchanges 1, 2, \ldots, \( n \). Let \( t^*_j \) be the entrepreneur who is just indifferent between listing on Exchange \( j \) or on Exchange \( j + 1 \), for \( j \in [1, n - 1] \). Furthermore, we set \( t_0 = \bar{t} \). Proceeding as in Lemma 5, we obtain:

\[
t^*_j(F_j, F_{j+1}) = 4\sqrt{\frac{F_j - F_{j+1}}{N(q_j - q_{j+1})}} \quad \text{for} \quad j \in [1, n - 1].
\]

The expected profit of Exchange \( j \) is:

\[
\Pi_j(F_j, F^*_{-j}) = \frac{(t_{j-1} - t_j)F_j}{\sigma} + \frac{\gamma N}{16\sigma}(t_j^2 - t_j^2).
\]

If the \( n \) exchanges obtain strictly positive expected profits then \( F^*_j \) is an interior solution of Exchange \( j \)’s maximization problem. Therefore it is solution of Exchange \( j \)’s first order condition, which is (for \( 1 < j \leq (n - 1) \)):

\[
\left( \frac{\partial t_{j-1}}{\partial F_j} - \frac{\partial t_j}{\partial F_j} \right) F_j + \left( t_{j-1} - t_j \right) + \frac{\gamma N}{8} \left( \frac{\partial t_{j-1}}{\partial F_j} t_{j-1} - \frac{\partial t_j}{\partial F_j} t_j \right) = 0.
\]

This is equivalent to:

\[
(t_{j-1} - t_j) - \frac{\partial t_j}{\partial F_j} (F_j - F_{j+1}) + \left[ -\frac{\partial t_j}{\partial F_j} \left( F_{j+1} + \frac{N\gamma t_j}{8} \right) + \frac{\partial t_{j-1}}{\partial F_j} \left( F_j + \frac{N\gamma t_{j-1}}{8} \right) \right] = 0.
\] (38)

Note that \( \frac{\partial t_j}{\partial F_j} > 0 \) and that \( \frac{\partial t_{j-1}}{\partial F_j} < 0 \). Furthermore, since \( t_{j-1} > t_j \):

50
\[ F_j + \frac{\gamma^N t_{j-1}}{8} > F_j + \frac{\gamma^N(t_j + t_{j-1})}{16} \quad \forall j > 1. \]

The R.H.S of this inequality is the per listing profit of Exchange \( j \) and therefore must be positive. Therefore the term in bracket in Equation (38) is negative. It follows that Equation (38) holds iff:

\[ (t_{j-1} - t_j) - \frac{\partial t_j}{\partial F_j} (F_j - F_{j+1}) > 0. \]  

(39)

Now, note that:

\[ \frac{\partial t_j}{\partial F_j} (F_j - F_{j+1}) = \frac{t_j}{2}. \]

Hence Equation (39) implies:

\[ t_{j-1} > \frac{3}{2} t_j \quad 1 < j \leq n - 1 \]  

(40)

Using the same reasoning, it can be checked that this inequality also holds for \( j = 1 \) (that is \( \bar{t} > \frac{3}{2} t_1 \)). Furthermore, note that \( t_{n-1} > \bar{t} \) if \( n \) exchanges are formed. Therefore, using a recursive argument, we deduce from Equation (40) that if \( n \) exchanges are formed profitably (attracts listings) then:

\[ \bar{t} > (3/2)^{(n-1)} \bar{t} \]

Using the fact that \( \bar{t} = m + \frac{\sigma}{2} \) and \( \bar{t} = m - \frac{\sigma}{2} \), this inequality turns out to be equivalent to \( \sigma > \sigma(n) \). This means that if \( \sigma \leq \sigma(n) \), there cannot be an equilibrium in which more than \( n \) exchanges are formed and obtain strictly positive expected profits. Therefore at most \( (n - 1) \) exchanges are formed if \( \sigma \leq \sigma(n) \). \( \Box \)
Figure 1: Timing of the Model

- **t=1**: Exchanges choose their trading technologies.
- **t=2**: Exchanges choose their listing fees.
- **t=3**: Entrepreneurs decide (i) where to list (ii) the issue size and (iii) the IPO price. Investors buy shares in the IPOs.
- **t=4**: Investors are hit by liquidity shocks and secondary market trading takes place.
- **t=5**: Firms’ payoffs are realized.
Figure 2

Demand for Shares in the IPO

\[ C_j = 0 \]

\[ C_j > 0 \]

\[ C_j' > C_j \]

\[ 0 \]

\[ 1 \]

\[ (1 - \infty) \]